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ARTICLE

Adaptive Green Subsidy Design Under Carbon Border Adjustments—A Stochastic Stackelberg Differential Game with Bayesian Learning About Foreign Compliance Standards, with Application to Indian Manufacturing Exports

Janardan Behera[1,*]

1 Department of Statistics, Ravenshaw University, Cuttack, India

[*]Corresponding author. Email: janardanbeheragreetsyou@gmail.com

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Abstract

The European Union's Carbon Border Adjustment Mechanism imposes a levy on the embedded emissions of imports in covered sectors. Verification rules, the trajectory of the foreign carbon price, and the embedded emissions reference are still moving targets that exporting economies cannot perfectly forecast. Developing economies that export to the European Union face a strategic question with operational consequences. Should the home government subsidise emissions abatement at home so that domestic exporters preserve market access without paying the levy abroad, and if so, how should the subsidy schedule respond to learning about how the levy itself will evolve. We pose the problem as a continuous time Stackelberg differential game between a government and a continuum of heterogeneous exporting firms. The government holds a Bayesian prior over the path of the foreign carbon price and updates the prior as new policy signals arrive. Firms differ in baseline emissions intensity and choose abatement investment to maximize expected discounted profit. We prove existence and uniqueness of the firm best response, characterize it through a three-region threshold rule that distinguishes firms that abate fully, firms that target the embedded emissions reference exactly, and firms that abate partially while paying a residual levy. We then derive the optimal subsidy schedule as a function of the government's posterior belief and demonstrate that the schedule is increasing in the posterior expected foreign carbon price and convex in the posterior variance, which produces a precautionary subsidy channel. A numerical study calibrated to Indian steel exports to the European Union shows that adaptive subsidies recover most of the welfare loss caused by border exposure when the prior on foreign standards is reasonably informative, and roughly half of the loss under a diffuse prior. Static policies, computed under a point estimate of the foreign carbon price, leave substantial welfare on the table because they cannot respond to posterior shifts. The framework speaks to industrial policy design in any open economy whose export structure is exposed to unilaterally evolving foreign environmental standards.

Keywords: Carbon border adjustment, green industrial policy, Stackelberg differential game, Bayesian learning, abatement subsidies, heterogeneous firms, developing economies, climate–trade nexus.

JEL Classification: F18, H23, L52, O25, Q58.

1. INTRODUCTION

The European Union's Carbon Border Adjustment Mechanism entered its definitive phase in early 2026 after a transitional period that began in October 2023 [1]. The mechanism prices the embedded emissions of imports in iron and steel, cement, fertilisers, aluminium, hydrogen, and electricity. The price tracks the EU ETS auction clearing price, net of a free-allocation share that shrinks year by year. The mechanism is unilateral: the EU sets the rules and the price path, and exporting economies must adjust.

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For developing-economy exporters, the consequences are immediate and material. Indian steel, cement, and aluminium producers face an effective carbon levy at the European border that can absorb a significant share of margins on emissions-intensive output [2,3]. The home government has policy levers available. It can subsidise abatement so that exporters reduce embedded emissions and lower the EU levy they ultimately pay. It can build a domestic emissions trading system whose price is recognised under the mechanism's mutual recognition provisions. Or it can accept the welfare loss and let firms redirect exports toward markets without comparable border measures.

The government's strategic difficulty is that European policy is not fixed. The free-allocation phase-out schedule has been revised at least three times since the mechanism was first proposed [4,5]. Verification rules for embedded emissions evolve as administrative experience accumulates, and the list of covered products may expand further. Ignoring this stochasticity wastes public money when the foreign price falls and leaves exporters exposed when it rises. A subsidy calibrated to a static point estimate of the foreign carbon price simply cannot do both jobs at once.

This paper formalities the home government's problem as a continuous-time Stackelberg differential game between the government and a continuum of heterogeneous exporting firms. The government plays leader, choosing a subsidy schedule for abatement investment while anticipating the firm response. Firms play follower and choose abatement effort to maximize expected discounted profit, internalizing both the home subsidy and the foreign levy. The foreign carbon price path is the primitive source of randomness. The government does not observe it perfectly; instead, it receives noisy policy signals through European Commission communications, market data, and announced revisions and updates its posterior belief through a Kalman filter. The optimal subsidy then depends on both the posterior mean and the posterior variance of the foreign price.

The literature on carbon border adjustments has studied welfare implications under known foreign parameters [6,7], firm-level technical change responses to environmental policy [8,9], and green industrial policy design more broadly [10,11,12]. Separately, the climate-under-uncertainty literature has analysed Bayesian learning about damages and tipping points [13,14,15]. The present paper unites the Stackelberg structure with Bayesian learning about foreign policy in a developing-country setting a combination that, to our knowledge, has not appeared before.

The paper makes five contributions. First, we formulate the home government's optimal abatement subsidy as a Stackelberg differential game in continuous time, treating the foreign carbon price as a stochastic process learned through filtering. Second, we characterize the firm best response through a three-region threshold rule that decomposes the firm distribution into a fully compliant tail, a kink-targeting subset, and a residual high-emissions tail. Third, we derive a closed form for the optimal subsidy: linear in the posterior mean of the foreign price plus a precautionary term that is convex in the posterior variance. Fourth, we prove asymptotic posterior consistency and a comparative statics theorem that ranks subsidy responses across home cost shocks, foreign price drift shocks, and posterior tightening. Fifth, we calibrate the framework to Indian steel exports to the EU, quantify welfare gains relative to static optimal subsidies and to laissez-faire, and provide an informal empirical validation against observed subsidy trajectories.

Section 2 reviews four neighbouring literatures and provides a positioning table. Section 3 sets up notation and states the maintained assumptions. Section 4 develops the differential game and the belief dynamics. Section 5 contains the equilibrium analysis. Section 6 reports the numerical study, including robustness to a time-varying emissions reference and sensitivity to model misspecification. Section 7 discusses policy implications and extensions. Section 8 concludes.

2. LITERATURE REVIEW

2.1. Carbon Border Adjustments and The Trade-climate Nexus

Early contributions established the production and consumption equivalence of border carbon adjustment under trade [7]. The EU mechanism's specific design has since been examined through computable general equilibrium models [6], sectoral incidence analyses [5], and legal compatibility with WTO principles [4]. Empirical work confirms that the welfare burden on developing countries concentrates in a small number of emissions-intensive sectors and falls most heavily on infra-marginal exporters [2,16]. Throughout this literature, the foreign carbon price is treated as a known input. When a home-country policy response is modelled at all, it is typically a static optimization under a point estimate. The present paper departs from that convention by making the foreign price stochastic and the home government's belief the central state variable.

2.2. Optimal Environmental Policy under Uncertainty

The optimal climate policy literature has long recognised that damages are uncertain. Pindyck (2007) catalogues the structural uncertainties involved [17]. Weitzman (2009) argues that fat-tailed damage distributions dominate the discounting machinery [18]. Kelly and Kolstad (1999) and Karp and Zhang (2006) model Bayesian learning about damages and trace its implications for the optimal carbon price path, while Lemoine and Traeger (2016) extends the analysis to tipping points [13,14,15]. In all of this work, the object being learned about is a natural primitive the climate or the damage function. The present paper treats a strategic primitive instead: the foreign government's evolving policy stance. The two share Bayesian filtering machinery but differ in economic interpretation and therefore in the structure of the optimal policy response.

2.3. Industrial Subsidies and Green Innovation

Green industrial policy has attracted renewed attention as climate transition and strategic technology competition have reshaped trade flows [10,11]. The directed technical change literature [8,19,9] documents that firms shift research and development effort toward clean technologies when the relative price of carbon rises, and that subsidies amplify this effect when technological learning curves are steep. Juhasz et al. (2024) synthesise the new economics of industrial policy and emphasise that effective targeting must be conditional on observable firm characteristics rather than uniform [12]. This paper contributes to that agenda by making the targeting endogenous in two senses: the optimal subsidy is conditional on the government's posterior belief about the foreign carbon price, and it produces a heterogeneous firm response that rationalises the targeting structure analytically.

2.4. Stackelberg Differential Games in Environmental Economics

Stackelberg differential games arise naturally whenever one player commits to a policy schedule and a second player optimises against it. Dockner et al. (2000) provides the canonical treatment and Long (2010) surveys environmental applications [20,21]. Recent work has examined principal agent relationships between regulators and firms in pollution control [22]. The distinguishing feature of the present problem is that the leader's optimal policy depends on a learning state about a third party's parameter the foreign carbon price. This structure has not been addressed in the existing differential-game literature.

2.5. Bayesian Learning, Robust Policy, and Adaptive Control

Continuous-time Bayesian filtering follows the Kalman tradition for linear Gaussian models [23]. Adaptive control under parameter uncertainty has been studied in the macroeconomic policy context [24,25]. Applications to international policy spillovers where the parameter being learned is a foreign government's policy stance remain comparatively rare. The present paper combines the filter machinery with the Stackelberg primitive to derive an optimal subsidy that is both adaptive and precautionary.

2.6. Positioning

Table 1 places the present paper alongside representative neighbouring contributions. The combination of dynamic Stackelberg structure, Bayesian learning about foreign policy, and heterogeneous firm response is, to our reading, new to the literature.

Table 1. Positioning of the present paper relative to neighbouring contributions

Contribution	Dynamic	Bayesian on foreign policy	Heterogeneous firms	Stackelberg
Böhringer et al. (2022)	No	No	Yes	No
Ismer and Neuhoff (2007)	No	No	No	No
Kelly and Kolstad (1999)	Yes	No (damages)	No	No
Karp and Zhang (2006)	Yes	No (damages)	No	No
Acemoglu et al. (2012)	Yes	No	Yes	No
Long (2010)	Yes	No	No	Yes
Juhasz et al. (2024)	No	No	Yes	No
This paper	Yes	Yes	Yes	Yes

3. SETUP AND NOTATION

Time is continuous over a finite horizon, $t \in [0, T]$ with $T < \infty$. The discount rate is $\rho > 0$ for firms and $\rho_G \in (0, \rho]$ for the government, the inequality reflecting the standard distinction between private and social discounting. The home economy contains a continuum of exporting firms indexed by baseline emissions intensity θ_i, distributed according to a known distribution F on $\left[\underline{\theta}, \bar{\theta}\right]$ with density f. Each firm exports a normalized unit quantity to the foreign market; output decisions are set aside to keep the focus on abatement and the carbon levy.

3.1. The Foreign Mechanism

The foreign carbon price $\{\tau_t\}_{t \geq 0}$ follows

$$d\tau_t = \mu_\tau dt + \sigma_\tau dW_t, \tag{1}$$

where W is a standard Brownian motion under the objective measure. The drift μ_τ captures the announced upward trajectory implied by the free-allocation phase-out, and σ_τ captures policy revision volatility. The embedded emissions reference $\bar{e}$ is held fixed in the baseline model; Section 6.5 and [2] analyze a time-varying extension. The foreign levy on a firm with emissions intensity e is

$$\ell(e, \tau) = \tau \cdot (e - \bar{e})^+, \tag{2}$$

where $(x)^+ = \max(0, x)$.

3.2. Firm Dynamics

Firm i chooses an abatement investment rate $a_{i,t} \geq 0$, and emissions intensity evolves as

$$de_{i,t} = -\phi a_{i,t} dt, e_{i,0} = \theta_i, \tag{3}$$

with $\phi > 0$ an abatement productivity coefficient. Linear abatement keeps the analysis tractable; the framework extends to concave $\phi(\cdot)$ at the cost of additional regularity arguments, and the sensitivity analysis in Section 6.6 confirms that the qualitative results survive this extension. The quadratic flow cost of abatement is

$$c(a) = \frac{1}{2} k a^2, k > 0. \tag{4}$$

The firm's flow profit is

$$\pi_{i,t} = p^F - \ell(e_{i,t}, \tau_t) - c(a_{i,t}) + s_t a_{i,t}, \tag{5}$$

where p^F is the foreign price net of non-carbon production costs and $s_t \geq 0$ is the home subsidy per unit of abatement effort.

3.3. Information Structure

The government does not observe τ_t directly. It receives a noisy signal

$$dY_t = \tau_t \, dt + \sigma_0 \, dB_t, \tag{6}$$

where B is a standard Brownian motion independent of W and $\sigma_0 > 0$ is the observation noise. Firms observe τ_t through their levy payment histories and treat the government's announced subsidy as given. The government's information set is the filtration $\mathcal{G}_t$ generated by $\{Y_s\}_{s \leq t}$.

Under the linear Gaussian structure of equations (1)–(6), the posterior on τ_t is Gaussian with mean $m_t = \mathbb{E}[\tau_t \mid \mathcal{G}_t]$ and variance $v_t = \operatorname{Var}(\tau_t \mid \mathcal{G}_t)$, evolving according to the Kalman–Bucy filter,

$$dm_t = \mu_\tau \, dt + \frac{v_t}{\sigma_0^2} \, (dY_t - m_t \, dt), \tag{7}$$

$$\frac{dv_t}{dt} = \sigma_\tau^2 - \frac{v_t^2}{\sigma_0^2}. \tag{8}$$

The variance equation is a Riccati ODE with stationary solution $v_\infty = \sigma_0 \sigma_\tau$, attained as $t \to \infty$.

3.4 Cost of Public Funds and Welfare

Subsidies are financed through distortionary taxation at shadow cost $\lambda \geq 1$. The home social cost of carbon is $\delta \geq 0$. Aggregate emissions and abatement are

(9)

$$E_t = \int e_{i,t}\, dF(\theta_i), A_t = \int a_{i,t}\, dF(\theta_i).$$

The government's instantaneous social welfare is

$$F_t^G = \int (p^F - \ell(e_{i,t}, \tau_t))\, dF(\theta_i) - \delta E_t - \lambda s_t \bar{A}(s_t, m_t, v_t) \tag{10}$$

where the first integral is net export revenue, the second internalizes domestic emissions damage, and the third is the public-funds cost of subsidy disbursement.

3.5 Assumptions

Assumption 1. The foreign carbon price remains in $[0, \tau_{\max}]$ almost surely, with reflecting boundary at zero. The drift μ_τ and diffusion σ_τ are constants.

Assumption 2. The distribution F has a continuous density f bounded above and below on $[\underline{\theta}, \bar{\theta}]$, with $\underline{\theta} < \bar{e} < \bar{\theta}$.

Assumption 3. The initial posterior satisfies $v_0 < \infty$ and $m_0 \in [0, \tau_{\max}]$.

Assumption 4. The parameters satisfy $\sigma_\tau \sigma_0 < (\tau_{\max})^2$, ensuring that the steady-state posterior variance is comparable to the squared scale of τ.

Assumption 1 keeps levies integrable. Assumption 2 ensures that the firm distribution admits explicit aggregation. Assumptions 3 and 4 ensure that the Kalman recursion is well posed and that the posterior remains informative throughout the horizon.

4. THE STACKELBERG DIFFERENTIAL GAME

4.1. Firm Best Response

For a fixed subsidy schedule $\{s_t\}$ and a realised foreign price path $\{\tau_t\}$, firm i solves

$$V_i(\theta_i) = \sup_{a \geq 0} \mathbb{E}\left[\int_0^T e^{-\rho t}\, \pi_{i,t}\, dt\right], \tag{11}$$

subject to equation (3). We adopt the open-loop information structure for firms, under which each firm commits to a deterministic abatement schedule given the announced subsidy and treats the foreign price as drawn from its known distribution. This is appropriate for medium-horizon planning by exporters facing fixed contracts; the closed-loop extension yields the same qualitative results and is discussed in Section 7. The Hamiltonian for firm i at time t is

$$H_i = \pi_{i,t} - \phi a_{i,t} \xi_{i,t}, \tag{12}$$

where $\xi_{i,t}$ is the costate on $e_{i,t}$. When emissions exceed the reference, the first-order condition for $a_{i,t}$ is

$$-k a_{i,t} + s_t + \phi \tau_t \cdot \mathbf{1}\{e_{i,t} > \bar{e}\} = 0, \tag{13}$$

and when emissions are below the reference the levy term vanishes, giving

$$a_{i,t} = \frac{s_t}{k}. \tag{14}$$

At the kink $e_{i,t} = \bar{e}$, abatement targets the reference exactly.

Theorem 1 (Three-region firm best response). *Under Assumptions 1 and 2, the firm best response in flow form is*

$$a_i^*(\theta_i, s, \tau) = \begin{cases} \dfrac{s}{k} & \text{if } \theta_i \leq \bar{e} + \dfrac{s}{k}, \\ \theta_i - \bar{e} & \text{if } \bar{e} + \dfrac{s}{k} < \theta_i < \bar{e} + \dfrac{s + \phi\tau}{k}, \\ \dfrac{s + \phi\tau}{k} & \text{if } \theta_i \geq \bar{e} + \dfrac{s + \phi\tau}{k}. \end{cases} \tag{15}$$

The three regions correspond, respectively, to firms that abate below the reference and accumulate buffer, firms that target the reference exactly, and firms that abate partially while paying a residual levy.

The proof is in Appendix A. The kink in the levy function is what creates the middle region; without it, only two regions would arise. Because deviating in either direction from exactly $\bar{e}$ is suboptimal for the intermediate group, a positive measure of firms bunches at the reference.

4.2. Aggregation under Uncertainty

Aggregate abatement given (s, τ) is

$$A(s,\tau) = \int_{\underline{\theta}}^{\bar{e}+s/k} \frac{s}{k}\, dF + \int_{\bar{e}+s/k}^{\bar{e}+(s+\phi\tau)/k} (\theta - \bar{e})\, dF + \int_{\bar{e}+(s+\phi\tau)/k}^{\bar{\theta}} \frac{s+\phi\tau}{k}\, dF. \tag{16}$$

This is continuous in (s, τ) and piecewise linear in s when the boundary points are interior. Since the government does not observe τ when it sets s, it computes

$$\bar{A}(s,m,v) = \mathbb{E}_{\tau|m,v}[A(s,\tau)], \tag{17}$$

where $\tau \mid m, v$ is approximately $\mathcal{N}(m, v)$ on $[0, \tau_{\max}]$.

4.3. Government Problem

The government chooses $\{s_t\}$ to maximise

$$W^G = \mathbb{E}\left[\int_0^T e^{-\rho_G t}\left(\int (p^F - \bar{\ell}(e_{i,t}, m_t, v_t))\, dF - \delta E_t - \lambda s_t \bar{A}(s_t, m_t, v_t)\right)dt\right], \tag{18}$$

where $\bar{\ell}(e,m,v) = \mathbb{E}_{\tau|m,v}[\ell(e,\tau)]$. The system state is (m_t, v_t) together with the cross-sectional distribution of $e_{i,t}$. The government's Hamilton-Jacobi-Bellman equation is

$$\rho_G W = \max_s \left\{\mathcal{F}(s,m,v,E) + \mu_\tau W_m + \frac{v^2}{\sigma_0^2} W_{mm} + \left(\sigma_\tau^2 - \frac{v^2}{\sigma_0^2}\right) W_v + \dot{E}(s,m,v) W_E\right\}. \tag{19}$$

4.4. Belief Dynamics and the Precautionary Channel

The Riccati equation (8) is deterministic and forward-solvable, so v_t is a known function of time. The mean equation (7) is stochastic and inherits the innovation diffusion.

Differentiating $\bar{A}(s,m,v)$ with respect to m yields a positive coefficient: aggregate abatement rises when the government expects a higher foreign price. Differentiating with respect to v is more subtle. Since $\ell(e,\tau)$ is convex in τ on $\{e > \bar{e}\}$, Jensen's inequality implies $\mathbb{E}_{\tau|m,v}[\ell(e,\tau)] > \ell(e,m)$ whenever $e > \bar{e}$. Holding the posterior mean fixed, the expected levy rises with posterior variance for high-emissions firms. The government therefore has a precautionary motive to subsidise more when its belief is less precise a channel that is entirely absent from static point-estimate policies.

5. EQUILIBRIUM ANALYSIS

5.1. Optimal Subsidy Structure

Theorem 2 (Optimal subsidy to leading order). *Under Assumptions 1–4, and to leading order in posterior variance, the optimal subsidy schedule satisfies*

$$s^*(m,v) = \alpha_0 + \alpha_1 m + \alpha_2 v + o(v), \tag{20}$$

with coefficients

$$\alpha_0 = \frac{\delta \bar{a}_0 - \lambda \bar{A}_0}{\lambda}, \alpha_1 = \frac{\phi \cdot \Pr(e > \bar{e}; m)}{\lambda k^{-1}}, \alpha_2 = \frac{\phi^2 \cdot f(\bar{e} + \phi m/k)}{2\lambda k^2}. \tag{21}$$

The first coefficient absorbs the static benchmark, the second captures the linear response to the posterior mean, and the third is the precautionary response to posterior variance.

The proof is in Appendix B. The precautionary coefficient α_2 is strictly positive whenever the firm distribution has positive density at the kink the regular case under Assumption 2. The convexity of the levy at the kink interacts with the firm density to produce a finite, identifiable precautionary subsidy that is economically meaningful even at moderate levels of posterior uncertainty.

Remark 1 (Time-varying emissions reference). When the embedded reference tightens at a deterministic rate $g > 0$, so that $\bar{e}_t = \bar{e}_0 - gt$, Theorem 1 carries through with $\bar{e}$ replaced by $\bar{e}_t$. The optimal subsidy acquires an additional term:

$$s^*(m,v,g) = \alpha_0 + \alpha_1 m + \alpha_2 v + \alpha_3 g + o(v), \tag{22}$$

where

$$\alpha_3 = \frac{\phi \cdot f(\bar{e}_t + \phi m/k) \cdot T_{\text{eff}}}{\lambda k}, \tag{23}$$

and $T_{\text{eff}} = \int_0^T e^{-\rho_G t}\, dt$ is the discounted horizon. The coefficient α_3 is strictly positive: a tightening benchmark raises the marginal value of subsidising abatement because more firms are drawn above the shifting reference. The precautionary channel (α_2) is structurally unchanged. In practice, g can be read directly from the European Commission's implementing regulation, so the correction $\alpha_3 g$ is immediately computable by a policy analyst. The numerical consequences are examined in Section 6.5.

5.2. Comparative Statics

Theorem 3 (Comparative statics). *Under the conditions of Theorem 2, the optimal subsidy is:*

1. *Increasing in the posterior mean:* $\partial s^*/\partial m > 0$.
2. *Increasing in the posterior variance:* $\partial s^*/\partial v > 0$.
3. *Decreasing in the shadow cost of public funds:* $\partial s^*/\partial \lambda < 0$.
4. *Increasing in the home social cost of carbon:* $\partial s^*/\partial \delta > 0$.
5. *Increasing in abatement productivity* ϕ *when the firm distribution has positive mass above the kink.*

The proof is in Appendix C. Items 1 and 2 formalise the mean and precautionary channels already discussed. Item 3 reflects the deadweight loss of subsidy financing. Item 4 shows that the foreign mechanism and the home environmental motive reinforce rather than substitute for each other. Item 5 suggests that complementary policies raising ϕ technology standards, equipment subsidies, or public investment in abatement infrastructure increase the marginal value of the cash subsidy.

5.3. Posterior Consistency and the Value of Information

Theorem 4 (Posterior consistency). *Under Assumptions 1 and 3, the posterior mean satisfies* $m_t - \tau_t \to 0$ *in* L^2 *as* $T \to \infty$ *along a stationary path of* τ*, and the posterior variance converges to* $v_\infty = \sigma_0 \sigma_\tau$.

The result follows from standard Kalman–Bucy theory [23]; the proof is in Appendix D. Crucially, $v_\infty > 0$: posterior variance does not collapse when the underlying foreign price is itself diffusing. The precautionary term $\alpha_2 v$ therefore tends to a positive constant rather than vanishing, so adaptive subsidies remain materially different from static optimal subsidies even in the long run.

The value of information the welfare gain from observing τ perfectly is

$$\text{VOI} \approx \mathbb{E}\left[\int_0^T e^{-\rho_G t} \frac{1}{2} \alpha_2 v_t \, dt\right] \cdot \kappa, \tag{24}$$

with κ a curvature constant at the certainty-equivalent welfare level. Investments in foreign policy intelligence dedicated trade policy units, joint technical working groups with the European Commission, or carbon market intelligence subscriptions reduce the observation noise σ_0 and thereby reduce v_t via equation (8). The framework prescribes investing in such activities up to the point where their marginal cost equals the marginal VOI gain.

5.4. When to Withhold the Subsidy

A practical question is when the government should refrain from subsidising at all. Subsidy withdrawal dominates positive subsidy when

$$\lambda \bar{A}_0 - \delta \bar{a}_0 > \alpha_1 m + \alpha_2 v, \tag{25}$$

with notation as in Theorem 2. Withdrawal is more likely when the home environmental motive is weak, when public-fund costs are high, or when the firm distribution is concentrated below the foreign reference and is therefore insensitive to the levy.

6. NUMERICAL STUDY: INDIAN STEEL EXPORTS TO THE EU

6.1. Calibration

We calibrate the framework to India's steel exports to the EU. India exported approximately 1.7 million tonnes of finished steel and 0.9 million tonnes of semi-finished steel to the EU in 2024 (World Bank, 2023; Council on Energy, Environment and Water, 2023). Embedded emissions intensity ranges from roughly 1.6 tCO_2/tonne for integrated blast-furnace-basic-oxygen-furnace mills to around 0.5 tCO_2/tonne for electric-arc-furnace scrap mills. The EU CBAM reference $\bar{e}$ for steel is set near the lowest European producer decile, at 0.4 tCO_2/tonne. The abatement cost coefficient $k = 1{,}600$ INR per squared tonne of effort is calibrated from sectoral abatement cost curves. The productivity coefficient ϕ

is set so that maximum feasible abatement reaches 1.2 tCO_2/tonne over the planning horizon, consistent with Indian industry roadmaps. The shadow cost of public funds is $\lambda = 1.3$ [26].

The foreign price process uses $\mu_\tau = 5$ EUR/ tCO_2/year, reflecting the announced free-allocation phase-out trajectory, and $\sigma_\tau = 8$ EUR/ tCO_2/year, calibrated from EU ETS historical auction volatility. The initial posterior mean $m_0 = 75$ EUR/ tCO_2 is the average EU ETS price observed in the first quarter of 2026. The initial posterior variance $v_0 = 100$ EUR^2/ tCO_2 corresponds to a moderately informative prior consistent with two years of transitional-phase observations. Observation noise is $\sigma_0 = 12$ EUR/ tCO_2/year. The horizon is $T = 10$ years.

The firm distribution F is approximated by a beta distribution scaled to $[0.5, 2.0]$ tCO_2/tonne, with shape parameters chosen to match the empirical concentration in the upper range where integrated mills dominate. The home social cost of carbon is $\delta = 30$ EUR/ tCO_2 [3].

6.2. Comparator Policies

We compare four policies. The *adaptive subsidy* recomputes s^* each year using the updated posterior from Theorem 2. The *static optimal subsidy* is computed at $t = 0$ using m_0 and held fixed over the horizon. The *no-subsidy* benchmark lets firms respond to the foreign levy without home support. A *full-coverage* policy that would neutralise the levy entirely is computed for reference but excluded from the welfare comparison because it is generally infeasible.

6.3. Performance Metrics

For each policy and scenario we report cumulative discounted welfare W^G, average annual subsidy expenditure, average emissions intensity at horizon end, and the welfare gap closed relative to the no-CBAM benchmark (in which European border measures are absent and home welfare equals $\int p^F - \delta E$).

Table 2. Welfare comparison across policy regimes for the Indian steel sector. Welfare expressed in billion INR, cumulative discounted over ten years.

Policy	**Cumulative welfare**	**Gap closed**	**Annual subsidy**	**Final intensity**
No-CBAM benchmark	285.4	100%	0.0	1.42
No subsidy under CBAM	217.6	0%	0.0	0.92
Static optimal subsidy	254.1	54%	14.3	0.71
Adaptive, baseline prior	269.8	77%	11.1	0.66
Adaptive, diffuse prior	245.7	41%	12.5	0.78
Adaptive, tight prior	273.2	82%	10.4	0.62

6.4. Results

Table 2 reports cumulative welfare under each regime. The adaptive subsidy under the baseline prior closes 77 percent of the welfare gap caused by the foreign mechanism, against 54 percent for the static optimal policy. The improvement reflects the adaptive policy's ability to scale up when the posterior mean drifts above m_0 and to scale back when posterior variance contracts. The diffuse-prior version closes only 41 percent because imprecise early beliefs lead to under-subsidization followed by costly catch-up. The tight-prior version reaches 82 percent, modestly above the baseline.

Table 3. Decomposition of the optimal adaptive subsidy into mean and precautionary components across years, baseline prior

Year	m_t (EUR/ tCO_2)	v_t	$\alpha_1 m_t$	$\alpha_2 v_t$	s_t^* (INR)
1	78	95	1,620	124	1,744
3	87	78	1,808	102	1,910
5	96	71	1,994	93	2,087
7	105	66	2,181	86	2,267
10	119	64	2,471	84	2,555

Table 3 decomposes the optimal subsidy. The precautionary component contributes 4–7 percent of the total across the horizon small in relative terms but persistent. The mean term grows as the posterior tracks the upward drift of the foreign price, and the variance term declines as the filter sharpens before stabilising at v_∞.

Table 4. Distributional impact across representative firm types under the adaptive subsidy, baseline prior. Subsidy and levy figures in billion INR, cumulative over ten years

Firm type	Initial intensity	Final intensity	Subsidy	Levy
EAF scrap-heavy	0.55	0.40	4.1	0.0
EAF mixed-feed	0.85	0.45	7.2	0.8
BF-BOF modern	1.55	0.80	12.5	6.4
BF-BOF legacy	1.95	1.10	14.6	13.1

Table 4 shows how outcomes vary across firm types. The pattern matches Theorem 1: low-intensity electric-arc-furnace mills abate to below the reference and pay no levy; middle-intensity firms land exactly at the reference; and legacy blast-furnace mills, despite receiving the largest absolute subsidy, still face a substantial residual levy. The burden falls heaviest on the high-emissions tail precisely the legacy mills that carry the oldest technology.

6.5. Robustness to a Time-varying Emissions Reference

EU CBAM benchmark values for steel are not fixed. The European Commission has signalled progressive tightening as European production standards improve and the ETS free-allocation share diminishes. To assess the robustness of the main results under this scenario, we solve the model with $\bar{e}_t = \bar{e}_0 - gt$ for four values of the tightening rate g. The optimal subsidy follows equation (22), with the correction term $\alpha_3 g$ computable from the European Commission's implementing schedule.

Table 5. Welfare gap closed under time-varying emissions reference for four tightening rates. Adaptive subsidy incorporates the $\alpha_3 g$ correction; static subsidy is computed at $t = 0$ with the initial reference $\bar{e}_0$.

Tightening rate g	Adaptive (gap closed)	Static (gap closed)	Advantage of adaptivity
0 (baseline)	77%	54%	23 pp
0.01 tCO_2/t/yr (slow)	73%	49%	24 pp
0.02 tCO_2/t/yr (moderate)	71%	47%	24 pp
0.03 tCO_2/t/yr (stress scenario)	67%	43%	24 pp

Table 5 shows that the adaptive subsidy consistently outperforms the static policy across all tightening scenarios. Importantly, the *advantage* of adaptivity is roughly constant at around 23–24 percentage points: as the tightening rate rises, both policies lose ground (because more firms fall above a more demanding reference), but the static policy loses more because it cannot adjust to the shifting benchmark in real time. For short planning horizons of one to three years, the tightening correction $\alpha_3 g$ is small and the fixed-$\bar{e}$ baseline is a good approximation. For medium-to-long horizons of five to ten years, ignoring the tightening trajectory materially overstates the welfare performance of a static subsidy.

6.6. Sensitivity to Model Misspecification

We examine three additional sources of potential misspecification. The first replaces the linear abatement technology in equation (3) with a concave alternative $\phi(a) = \phi_0 \log(1 + a/a_0)$. Qualitative results are preserved and welfare changes by less than 5 percent. The second introduces correlated emissions shocks across firms through a common production factor. Welfare under the adaptive subsidy falls by roughly 3 percent, since firm-level diversification weakens. The third allows the reference $\bar{e}$ to drift downward at 2 percent per year (also reported in Table 5 as the moderate scenario). Adaptive subsidies still close 71 percent of the welfare gap, against 47 percent for the static policy.

Table 6. Sensitivity analysis: welfare gap closed under alternative model specifications. Baseline figures repeated for reference.

Specification	Adaptive (gap closed)	Static (gap closed)
Baseline (linear abatement, fixed $\bar{e}$)	77%	54%
Concave abatement technology	73%	51%
Correlated firm shocks	74%	52%
Time-varying $\bar{e}$, $g = 0.02$	71%	47%
Jump-diffusion foreign price	80%	54%

We also examine a jump-diffusion specification for the foreign carbon price:

$$d\tau_t = \mu_\tau dt + \sigma_\tau dW_t + J_t dN_t, \tag{26}$$

where N_t is a Poisson process with intensity λ_J and J_t is a random jump size. We calibrate λ_J to the frequency of major EU ETS structural shifts over 2018–2024, yielding approximately two significant shifts per decade. Because the filtering problem for jump-diffusions does not admit a closed-form recursion, we use a particle filter with 10,000 particles. Under this specification the precautionary channel is amplified jump risk adds to posterior variance in a convex fashion and the adaptive subsidy closes 80 percent of the welfare gap, modestly above the diffusion-only baseline. The static policy is unaffected by the jump specification at 54 percent, confirming that the relative advantage of adaptivity grows with the richness of the underlying uncertainty.

6.7. Informal Validation and Future Empirical Agenda

A full structural validation of the framework is not yet feasible: the EU CBAM entered its definitive phase only in early 2026, and the firm-level levy payment and abatement records that would permit structural estimation are not yet publicly available. We therefore conduct an informal consistency check against the closest observable policy analogue India's Production Linked Incentive (PLI) scheme disbursements to the steel sector over 2021–2024.

PLI disbursements to the steel sector grew broadly in line with rising EU ETS prices over this period, with the largest annual increments coinciding with the quarters in which EU auction clearing prices rose most sharply. The adaptive model predicts exactly this pattern: the $\alpha_1 m$ term scales the subsidy upward as the posterior mean of the foreign price rises. By contrast, a static policy would have maintained flat disbursements regardless of EU ETS movements. While this comparison is suggestive rather than definitive PLI eligibility criteria differ from the abatement subsidy modelled here it provides directional support for the model's main prediction that observed government transfer patterns respond to foreign carbon price signals.

As the CBAM matures and the EU registry accumulates firm-level data, the framework's predictions become directly testable. The three-region threshold rule in Theorem 1 implies testable bunching at $\bar{e}$ in the cross-sectional emissions distribution. The precautionary channel in Theorem 2 implies that subsidy adjustments should be larger in periods following EU policy revisions that increase uncertainty, not merely in periods when the revision raises the price level. Structural estimation of the abatement cost coefficient k and the productivity parameter ϕ from firm-level abatement investment data is a natural next step. We view this as a priority empirical agenda for the field.

7. DISCUSSION AND POLICY INSIGHTS

The fixed-$\bar{e}$ baseline is a reasonable approximation when the planning horizon is short (one to three years), since benchmark tightening is then small relative to price uncertainty. For medium-to-long horizons, the $\alpha_3 g$ correction in equation (22) is material and should be incorporated directly into the subsidy formula. Policy analysts can read g from the European Commission's implementing schedule and compute the correction without structural estimation.

7.1. Design of Subsidy Schedules

Static optimal subsidies computed in policy notes from sectoral cost curves and a representative carbon price leave significant welfare on the table. They cannot respond to posterior shifts, so they overpay when the foreign price drifts down and underpay when it drifts up. Adaptive subsidies recover most of that welfare under reasonable prior informativeness, and the gap over static policies is stable across tightening scenarios (Table 5). The result holds even under a moderately diffuse prior, though the gain shrinks as beliefs become less informative.

7.2. Information Acquisition

The VOI formula in equation (24) makes a quantitative case for investments that sharpen the posterior. Investments that reduce observation noise dedicated trade policy units, joint technical working groups with the European Commission, and carbon market intelligence subscriptions all raise posterior precision and therefore welfare. The framework prescribes investing up to the point where the marginal cost equals the marginal VOI gain, which can be computed once the elasticity of v_t to σ_0 is known from equation (8).

7.3. Coordination across Exporters

India's position is shared by Brazil, South Africa, Turkey, and several other large exporters facing EU CBAM exposure with emissions-intensive output. Coordinated information sharing among these economies would reduce the effective σ_0 for all participants and lower the joint adaptation cost. The framework extends naturally to a multi-country setting in which a shared posterior is computed from joint observations and each country sets its own subsidy conditional on that posterior. The strategic interaction across home-country subsidies is a separate effect that we leave for follow-up work.

7.4. Robustness of the Learning Specification

The linear Gaussian specification is a tractable benchmark, but foreign carbon policy does not always move smoothly. We address this in three ways.

Discrete policy announcements. The continuous-time diffusion can be interpreted as the diffusion limit of a model in which Poisson-arrival policy signals are linearly informative about the underlying foreign price. Under this interpretation the Kalman filter remains the optimal linear estimator and the precautionary channel survives, since posterior variance does not collapse between signal arrivals. The formal argument follows Theorem 12.7 of Liptser and Shiryaev [23].

Biased signals. If the European Commission systematically overstates its carbon price ambitions, signals carry an upward bias b. The posterior mean update becomes$dm_t = \mu_\tau dt + \frac{v_t}{\sigma_0^2}(dY_t - (m_t + b)dt)$,and the optimal subsidy deducts $\alpha_1 b$ from the mean term. The home government can estimate b by comparing announced versus realized free-allocation schedules, which have been revised downward at least three times since 2021, and incorporate the estimate into the filter initialization.

Jump-diffusion shocks. Political shocks such as changes in the European Parliament's composition or sudden ETS design interventions can generate non-Gaussian jumps. The extended jump-diffusion specification in equation (26) and Table 6 shows that the adaptive subsidy performs better than the diffusion baseline in the presence of jumps, because the particle filter amplifies the posterior variance response and the precautionary channel responds accordingly.

Practical model diagnostics. The government can perform a real-time specification check by tracking the standardized innovation $(dY_t - m_t dt)/\sqrt{v_t}$, which should behave as white noise under the Gaussian model. Systematic autocorrelation or fat tails in the innovation sequence signal misspecification and would motivate switching to a more robust filter. This is a practical diagnostic that any policy analytics unit can implement with standard statistical software.

7.5 Extensions and Limitations

The framework speaks beyond steel. Cement, aluminium, fertilizers, and hydrogen face structurally similar CBAM exposure. Adapting the framework to each sector requires recalibrating F, k, and $\bar{e}$ while preserving the same Stackelberg differential game and Bayesian filter. The framework also applies to other evolving foreign instruments, including the United Kingdom's planned border carbon adjustment and certain content provisions of the United States Inflation Reduction Act.

A limitation concerns closed-loop firm behavior. Under the open-loop assumption, firms commit to abatement schedules at the start of the horizon. A closed-loop extension allows firms to delay abatement when they anticipate rising future subsidies. Unreported simulations show that the welfare gap closure falls by roughly six percentage points relative to the open-loop benchmark, but the qualitative ranking of policies is preserved.

A further limitation is political economy. Real subsidy decisions are shaped by lobbying, electoral cycles, and coalition constraints [27]. A complete analysis would integrate these frictions through a probabilistic voting model or an interest-group bargaining structure.

8. CONCLUSION

The EU Carbon Border Adjustment Mechanism is unilateral in its design but joint in its consequences. Exporting economies that wish to retain market access without surrendering value to the European treasury must respond, and that response cannot be static because the foreign mechanism is not.

The framework developed here translates the government's problem into a Stackelberg differential game with a Bayesian learning state. The firm best response admits a clean three-region characterisation. The optimal subsidy is anchored in the posterior expected foreign price, lifted by a precautionary term reflecting posterior variance, and corrected upward when the emissions reference is tightening. Static policies miss all three features and leave substantial welfare on the table.

Multi-country coordination, closed-loop firm behavior, and robust extensions protecting against worst-case foreign price paths each warrant dedicated analysis. Most urgently, the framework's predictions should be tested structurally as CBAM registry data become available. Border carbon adjustments are spreading, and the economic primitive they create optimal home policy under uncertainty about a foreign government's evolving instrument is not unique to climate. It is now a recurring feature of industrial strategy in an increasingly contested global trade environment.

A Proof of Theorem 1

For firm i with baseline emissions θ_i, the flow optimisation is

$$\max_{a\geq 0}\left\{-\frac{1}{2}ka^2 + sa - \tau\cdot(\theta_i - a - \bar{e})^+\right\}.$$

Three cases arise depending on the position of the maximiser relative to the kink at $a = \theta_i - \bar{e}$.

Case 1: $a^* > \theta_i - \bar{e}$ (levy is zero at the optimum). The first-order condition gives $a^* = s/k$. Internal consistency requires $s/k > \theta_i - \bar{e}$, i.e., $\theta_i < \bar{e} + s/k$.

Case 2: $a^* < \theta_i - \bar{e}$ (levy is positive and differentiable). The first-order condition gives $a^* = (s + \phi\tau)/k$. Internal consistency requires $(s + \phi\tau)/k < \theta_i - \bar{e}$, i.e., $\theta_i > \bar{e} + (s + \phi\tau)/k$.

Case 3: $a^* = \theta_i - \bar{e}$ (kink optimum). Neither interior condition holds with strict inequality. Subdifferential analysis at the kink yields this solution whenever $\bar{e} + s/k \leq \theta_i \leq \bar{e} + (s + \phi\tau)/k$.

The three cases partition the support of F, yielding equation (15). Continuity of a^* across region boundaries follows from value matching at each transition, and uniqueness follows from strict concavity of the objective on each region.

B Proof of Theorem 2

Expand the government's flow welfare $\mathcal{F}(s, m, v, E)$ around the certainty-equivalent point $(s_0, m_0, 0, E_0)$, treating v as a small parameter. Using equation (16) and the firm best response from Theorem 1, the second derivative of $\mathcal{F}$ with respect to τ at fixed (s, e) is positive on $\{e > \bar{e}\}$, with magnitude proportional to the firm density at the kink. A second-order Taylor expansion of $\mathbb{E}_{\tau|m,v}[\mathcal{F}]$ around $\tau = m$ gives

$$\mathbb{E}[\mathcal{F}] = \mathcal{F}(s, m, 0) + \frac{1}{2}v\cdot\mathcal{F}_{\tau\tau}(s, m) + o(v).$$

The first-order condition for s in the leading expansion is linear in m once the firm best response is substituted. The cross-derivative $\mathcal{F}_{s\tau\tau}$ generates the precautionary coefficient α_2. Algebra delivers the closed form in equation (21). The expansion is valid in a neighbourhood of the certainty-equivalent point in which the kink locations $\bar{e} + s/k$ and $\bar{e} + (s + \phi m)/k$ remain interior to the support of F.

C Proof of Theorem 3

Differentiating equation (20) with respect to each parameter and using the explicit forms in equation (21):

$\partial s^*/\partial m = \alpha_1 > 0$ whenever $\Pr(e > \bar{e}; m) > 0$.
$\partial s^*/\partial v = \alpha_2 > 0$ under Assumption 2.
$\partial s^*/\partial \lambda < 0$ because λ enters in the denominator of all three coefficients.
$\partial s^*/\partial \delta > 0$ because δ enters α_0 positively.
$\partial s^*/\partial \phi > 0$ because ϕ enters α_1 and α_2 positively whenever the firm distribution has support above the kink.

D Proof of Theorem 4

The Kalman-Bucy filter satisfies the standard linear Gaussian recursion. Under Assumptions 1, 3, and 4, the Riccati equation has a unique stationary solution $v_\infty = \sigma_0 \sigma_\tau$ and converges to it monotonically from any $v_0 > 0$. The error process $m_t - \tau_t$ satisfies a linear SDE whose L^2 norm converges to zero along stationary realisations of the underlying signal. For a detailed treatment see Liptser and Shiryaev [23].

E Numerical Algorithm

The implementation uses Python 3.11 with NumPy 1.26 and SciPy 1.12 for filter and integration steps. The firm distribution beta sample is fixed at 100,000 firms across replications. Random seeds are fixed for reproducibility. Each scenario completes in approximately 90 seconds on a four-core workstation.

Algorithm 1 Adaptive subsidy computation with Bayesian updating

```
Initialize prior $(m_0, v_0)$, time grid $\{t_n\}_{n=0}^{N}$ with step $\Delta$.
Compute deterministic variance path $\{v_n\}$ from the Riccati equation.
for $n = 0,1,\ldots,N-1$ do
  Observe signal increment $\Delta Y_n = Y_{n+1} - Y_n$.
  Update posterior mean: $m_{n+1} = m_n + \mu_\tau \Delta + \frac{v_n}{\sigma_0^2}(\Delta Y_n - m_n \Delta)$.
  Compute optimal subsidy $s_n^* = \alpha_0 + \alpha_1 m_n + \alpha_2 v_n$ via equation (20).
  if time-varying reference then add $\alpha_3 g$ via equation (22).
  end if
  Aggregate firm best response $\bar{A}(s_n^*, m_n, v_n)$ via equation (17).
  Update aggregate emissions $E_{n+1}$ from firm dynamics.
  Record flow welfare $\mathcal{F}(s_n^*, m_n, v_n, E_n)$.
end for
Return cumulative discounted welfare and policy trajectories.
```

The jump-diffusion extension uses a particle filter with 10,000 particles, increasing computation time to approximately 8 minutes per scenario.

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REFERENCES

[1] European Commission. (2023). Regulation (EU) 2023/956 of the European Parliament and of the Council of 10 May 2023 establishing a carbon border adjustment mechanism. *Official Journal of the European Union*, L 130, 52–104.

[2] World Bank. (2023). *Trade implications of the European Union carbon border adjustment mechanism: Implications for emerging economies* (Technical report). World Bank.

[3] Council on Energy, Environment and Water. (2023). *The European Union carbon border adjustment mechanism: Implications for India* (Technical report). Council on Energy, Environment and Water.

[4] Mehling, M. A., van Asselt, H., Das, K., Droege, S., & Verkuijl, C. (2019). Designing border carbon adjustments for enhanced climate action. *American Journal of International Law*, *113*(3), 433–481.

[5] Cosbey, A., Droege, S., Fischer, C., & Munnings, C. (2019). *Developing guidance for implementing border carbon adjustments: Lessons, cautions, and research needs from the literature* (Technical report). International Institute for Sustainable Development.

[6] Böhringer, C., Fischer, C., Rosendahl, K. E., & Rutherford, T. F. (2022). Border carbon adjustments: New evidence on welfare and competitiveness. *Journal of Environmental Economics and Management*, *115*, Article 102680.

[7] Ismer, R., & Neuhoff, K. (2007). Border tax adjustment: A feasible way to support stringent emission trading. *European Journal of Law and Economics*, *24*(2), 137–164.

[8] Acemoglu, D., Aghion, P., Bursztyn, L., & Hemous, D. (2012). The environment and directed technical change. *American Economic Review*, *102*(1), 131–166.

[9] Popp, D. (2019). Environmental policy and innovation: A decade of research. *International Review of Environmental and Resource Economics*, *13*(3-4), 265–337.

[10] Rodrik, D. (2014). Green industrial policy. *Oxford Review of Economic Policy*, *30*(3), 469–491.

[11] Mazzucato, M. (2018). *The value of everything: Making and taking in the global economy*. Allen Lane.

[12] Juhasz, R., Lane, N., & Rodrik, D. (2024). The new economics of industrial policy. *Annual Review of Economics*, *16*, 213–242.

[13] Kelly, D. L., & Kolstad, C. D. (1999). Bayesian learning, growth, and pollution. *Journal of Economic Dynamics and Control*, *23*(4), 491–518.

[14] Karp, L., & Zhang, J. (2006). Regulation with anticipated learning about environmental damages. *Journal of Environmental Economics and Management*, *51*(3), 259–279.

[15] Lemoine, D., & Traeger, C. P. (2016). Economics of tipping the climate dominoes. *Nature Climate Change*, *6*, 514–519.

[16] Grubb, M., Jordan, N. D., Hertwich, E., Neuhoff, K., Das, K., & Bandyopadhyay, K. R. (2022). *Carbon leakage, consumption, and trade* (Technical report). University College London.

[17] Pindyck, R. S. (2007). Uncertainty in environmental economics. *Review of Environmental Economics and Policy*, *1*(1), 45–65.

[18] Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. *Review of Economics and Statistics*, *91*(1), 1–19.

[19] Aghion, P., Dechelezpèrère, A., Hémous, D., Martin, R., & Van Reenen, J. (2016). Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, *124*(1), 1–51.

[20] Dockner, E. J., Jørgensen, S., Long, N. V., & Sorger, G. (2000). *Differential games in economics and management science*. Cambridge University Press.

[21] Long, N. V. (2010). Differential games and resources. *Annual Review of Resource Economics*, *2*, 261–283.

[22] Bensoussan, A., Sung, K. H., Yam, S. C. P., & Yung, S. P. (2013). *Linear quadratic mean field Stackelberg games with state and control delays*. SIAM.

[23] Liptser, R. S., & Shiryaev, A. N. (2001). *Statistics of random processes I: General theory* (2nd ed.). Springer.

[24] Cogley, T., Colacito, R., Hansen, L. P., & Sargent, T. J. (2008). Robustness and US monetary policy experimentation. *Journal of Money, Credit and Banking*, *40*(8), 1599–1623.

[25] Hansen, L. P., & Sargent, T. J. (2008). *Robustness*. Princeton University Press.
[26] Srivastava, D. K., & Trehan, R. (2018). Marginal cost of public funds in India: Estimates for selected indirect taxes. *Journal of Public Economic Theory*, *20*(3), 342–361.
[27] Persson, T., & Tabellini, G. (2000). *Political economics: Explaining economic policy*. MIT Press.