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ARTICLE

Optimizing Antibiotic Selection for UTIs Using VIKOR and TOPSIS in an Atanassov Intuitionistic Fuzzy MCDM Framework

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Abstract

The measurement of information, along with the assessment of knowledge, plays a crucial role in the theory of Atanassov intuitionistic fuzzy sets (AInFSs). The primary objective of this manuscript is to explore the information and knowledge evaluation of AInF-sets and their application in decision-making (DMI) scenarios. This study introduces a novel knowledge quantification method for AInF-sets, addressing the shortcomings of existing information and knowledge assessment techniques. The reliability and efficiency of the proposed knowledge measurement approach are validated through numerical illustrations, comparing it with current methodologies within the AInF framework. Furthermore, leveraging the suggested knowledge evaluation, an accuracy assessment for AInF-sets is derived. The utilization of the proposed accuracy metric is demonstrated in pattern recognition challenges. To confirm its practical effectiveness, numerical case studies are provided. A refined version of the Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) technique, incorporating the proposed accuracy metric, is introduced to tackle a multi-criteria decision-making (MCDM) problem within an intuitionistic fuzzy setting. Finally, a real-world application is presented through a case study focused on selecting the most suitable antibiotic for treating urinary tract infections (UTIs). The efficiency of the suggested method is highlighted by comparing it with prevailing DMI strategies.

Keywords: Atanassov intuitionistic fuzzy set; Knowledge measure; Accuracy measure; Multi-Criteria Decision Making; VIKOR

1. INTRODUCTION

Numerous practical techniques and strategies have been developed by researchers to address uncertainty and impreci- sion in decision-making. In everyday life, decisions are made in every area. In an ideal world, every bit of information and knowledge has a unique value that clearly and uniquely describes it. Unfortunately, because of the unpredictability and complexity of practical application, the information we get is often inadequate, i.e., information with ambiguity [1–5]. This makes it a major difficulty to Figure out how to effectively understand ambiguous information to improve decision-making capability [6–8]. Probability theory was the sole instrument available to measure uncertainty and imprecision until Prof. [9]'s revolutionary discovery of fuzzy sets. Each element of the complete cosmos set in the unit interval is assigned a membership function by the fuzzy set [10–15] in order to specify the grades. However, hesitation degrees are present in many real-world situations, therefore the membership and non-membership functions in fuzzy sets are not mutually incompatible. Currently, other methods have been proposed to deal with this problem, such as intuitionistic fuzzy sets rough sets are found in [16–19]. Witness theory [20–22], R-number [23,18,24–28]. Among these, atanassov intuitionistic fuzzy sets (AInFSs), an extension of fuzzy sets (FSs), is particularly useful for managing uncertain data. The main distinction between AInFSs and FSs is that the former differentiate between

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an element's membership and non-membership grade, while the latter better captures human hesitancy. Consequently, AInFSs have gained widespread popularity and are being employed in several industries, such as pattern classification. Medical evaluations are presented in [29–33]. Information fusion is discussed in [34–39], and others [40–47]. For the entropy of the fuzzy set, which has been the subject of continuing study. Ever since fuzzy entropy was first introduced by [48], researchers have been captivated by it. [49] published the fuzzy entropy axiom and defined it using the Shannon function [50]. The measure of intuitionistic entropy was initially axiomatically developed by [51] and was only reliant on hesitation degree. Three main structures in entropy allow for uncertainty, hesitancy, intuition modelling, and the use of the Shannon entropy concept of probability and unreliability. [52–53]. A large number of researchers, including [54–56] focus on how an AInF-set's entropy is defined. Hence, rather than the entropy measure, the idea of the knowledge measure may be seen as a complementary concept for the total uncertainty measure [57]. Therefore, in this study, rather than focusing on the relationship between entropy and knowledge measure, we construct a new axiomatic framework inside the context of knowledge measure to address the problem that entropy cannot solve. Investigating the amount of knowledge that AInFSs convey, Szmidt et al. conducted a groundbreaking study [58–63]. The phrase "knowledge" refers to information that is consistent, accurate, and unique and is considered useful in a certain context. When addressing AInFSs, it is not sufficient to assume that entropy and knowledge measure possess a confident logical basis, according to [59,64–66]; rather, knowledge measure needs to be viewed from several sides. Many theories emphasise the inherent ambiguity of information content, even if certain notions support it [67]. The concepts discussed above imply that there isn't an axiomatic theory of knowledge measure that integrates information clarity and substance.

The most recent study results [68]emphasise and demonstrate that, at the very least, information content and information clarity are associated to AInFSs. [69] found that while solving multi-criterion decision-making (MCDM) issues, the knowledge measure was applied to calculate the weight of each and every attribute. Furthermore, [70] carried out a comprehensive analysis of the conceptual definitions of AInF-information measures. In an MCDM problem, we search the various options for a particular choice that meets the greatest number of predetermined criteria. There are numer- ous studies on this matter, including [71–77]. An MCDM problem's conclusion always includes a crucial term, such the criteria's weights. We may use the weights for the justified criterion to determine which choice is best. There are several ways to determine the weights of the criterion. In order to tackle MCDM challenges, [73] suggested the VIKOR methodology, which can offer a workable solution. This technique selects the best option based on a precise measure of "Closeness" to the ideal answer. Numerous works extended the traditional VIKOR approach to handle problems related to MCDM, MADM, and MCGDM. The fuzzy VIKOR approach was em- ployed by [78] to address the vendor selection issue. [79] looked into a case to determine which hospital in Taiwan was the best. For the purpose of selecting the plant's location, [80] extended the VIKOR method. [81] rated the medical professionals using the VIKOR approach. Using a basic AHP model, [82] developed a set of indicators to evaluate the iron and steel sector's long-term viability in Libya. As per [83], the order of Initial Public Offerings (IPOs) in India need to be determined by their respective performances. Most scientists used the distance metric to identify the VIKOR approach's maximum collective benefit and lowest individual sorrow. On the other hand, we use the provided similarity measure in the proposed approach, and the results are really beneficial. According to the aforementioned article, there is still room for debate over AInF-knowledge measurements. Differentiating between AInF-sets and its complementary is a major focus of a significant number of studies related to AInF-knowledge and information measures. This novel approach to investigating AInF-knowledge measures was developed by [67]; nevertheless, further research is required to refine it and provide a practical measure that will quantify every bit of knowledge of a given AInF-set. Important results from the study of AInF-information and knowledge measurements show that some problems in intuitionistic fuzzy settings cannot be fully handled by any one of these key discoveries. Here, we present a method for resolving MCDM problems with the use of suggested AInF- information and precision measures. Numerous helpful conclusions about AInF-information measures could not completely address the challenges associated with decision-making. The following are the driving forces for our decision to carry out this investigation:

(a) The majority of AInF-knowledge and information measures do not meet the requirements for linguistic analysis's required order. Conversely, as seen in Example 1, the suggested AInF-exponential knowledge measure attains preferred ranking.

(b) Most estimates of AInF-knowledge and information measures provided in the literature produce absurd results when assessing uncertainty between different AInF-sets (see Example 2).

(c) While the majority of AInF knowledge and information measures determine the same criteria weights for several substitutes, the proposed AInF-knowledge measure determines different criteria weights for multiple substitutes (see Example 3).

(d) Many similarity and dissimilarity measurements fail to find a pattern among the potential patterns in an intuitionistic fuzzy environment. But among the provided patterns, the proposed AInF-accuracy metric clearly identifies the pattern (see Example 4).

Based on these results, in this study, we proposed an effective AInF-knowledge measure that successfully addresses the limitations of existing measures reported in the literature. Our approach resolves issues related to ranking inconsistencies, uncertainty estimation, and pattern recognition in an intuitionistic fuzzy environment. Unlike previous measures that struggle with accurate attribute weight calculations, our proposed measure assigns distinct and meaningful weights to multiple alternatives, ensuring a more precise decision-making process. It also provides reliable ambiguity computations, avoiding the irrational or inconsistent results observed in prior studies. Additionally, the proposed measure enhances linguistic comparisons by maintaining the correct ranking order, making it more suitable for applications that rely on linguistic analysis. Furthermore, it significantly improves pattern recognition by effectively identifying patterns within an intuitionistic fuzzy environment, overcoming the challenges faced by traditional similarity and dissimilarity measures. Overall, the proposed AInF-knowledge measure enhances the accuracy and reliability of knowledge representation, making it a powerful and practical tool for decision-making in fuzzy environments. The major contributions of the current study are as follows:

(a) A knowledge measure in the AInF context is suggested in this paper. Its properties are thoroughly investigated.

(b) Some examples provide the shortcomings of various current information measures in the AInF context.

(c) The application of the proposed accuracy measures is given in pattern detection issues. The proposed accuracy measures are used to solve a numerical example of pattern detection issue and a comparison with other measures is also taken to find their effectiveness.

(d) Some numerical examples are provided to support the study's assumptions and conclusion

(e) A modified VIKOR strategy is provided for tackling an MCDM problem. In the suggested method, the proposed AInF-accuracy is used instead of the distance measure.

(f) The proposed VIKOR approach assists us in selecting the best antibiotic medicine to treat urinary tract infections (UTIs).

In accordance with the goal of the current investigation, the entire paper is organised as follows: Section 1summarises the goals of the study, its purpose, and the total contributions made by earlier scientists in the field. A few key definitions of the discipline are covered in Section 2. Section 3 covers the earlier research in the topic of AInF. This part assesses the validity of the suggested knowledge measure and defines it within the framework of AInF. Additionally, the merits of the proposed measure are discussed and contrasted with others in this section. Section 4 presents the development of an accuracy measure inside the AInF context using the suggested knowledge measure. Pattern detection problems make use of the suggested accuracy measure. A numerical example verifies their efficacy. The VIKOR technique is presented in Section 5 and is applicable to MCDM problems. A case study concerning the choice of the most effective antibiotic for treating urinary tract infections is solved using the suggested method. A representative comparison between the suggested technique and the well-known approaches is provided in this section. Finally, conclusions, flaws, and future directions for the entire study are included in Section 6.

2. PRELIMINARIES

Within the current section, We briefly review some basic knowledge on AInF-sets in order to facilitate the description that follows.
Let us assume that

$$\Psi_h = \{W = (w_1, w_2, w_3, \ldots w_h) | \sum_{i=1}^{h} w_i = 1 \; where \; 0 \leq w_i \leq 1 \forall i = 1,2,\ldots,h\}, \tag{1}$$

is the complete probability distribution accumulation for $w \geq 2$.
According to [50], the entropy measure is

$$C(W) =- \sum_{i=1}^{h} w_i \, log(w_i); \tag{2}$$

where $W \in \Psi_h$. Numerous methods have been used in the literature to show generalised entropies. Shannon entropy is useful in a wide range of industries, including computers, statistics, data mining, and finance.

[84] provided support for the Shannon entropy generalisation of order-p put forth by

$$C_p(W) = \frac{1}{1-p} \log\left[\sum_{i=1}^{h} (w_i)^p\right], p \neq \; 1, p > \; 0. \tag{3}$$

Exponential entropy was proposed by [85,86] another measure based on these considerations is given by

$$B_Y(W) = \sum_{i=1}^{h} w_i \left(e^{(1-w_i)} - 1\right). \tag{4}$$

Shannon's Entropy is pointed out be advantaged over by the exponential entropy by these authors. For example, with regard to uniform probability distribution $W = \left(\frac{1}{w}, \frac{1}{w}, \ldots, \frac{1}{w}\right)$ exponential entropy possesses fixed upper bound

$$\lim_{w\to\infty} B_Y\left(\frac{1}{w}, \frac{1}{w}, \ldots, \frac{1}{w}\right) = e - 1; \tag{5}$$

and that Shannon's entropy does not in this instance.

Definition 1. ([9]) Consider a finite set $J(\neq \phi)$. A fuzzy set $\bar{P}$ defined on J is given by

$$\bar{P} = \{ < j_i, \mu_{\bar{P}}(j_i) > : j_i \in J\}; \tag{6}$$

where $\mu_{\bar{P}} : J \rightarrow [0, 1]$ represents a membership function for $\bar{P}$.

Definition 2. ([16]) Consider a finite set $J(\neq \phi)$. An AInF-set P defined on J is given by

$$P = \{ < j_i, \mu_P(j_i), \nu_P(j_i) > : j_i \in P\}; \tag{7}$$

where μ_P : J → [0,1] and ν_P : J → [0,1] are membership degree and non-membership degree respectively, with the condition

$$0 \leq \mu_P(j_i) + \nu_P(j_i) \leq i, \forall j_i \in J. \tag{8}$$

The hesitation degree of AInF-set P defined in J is denoted by (π_P) $\forall \, j_i \in J$, and to compute the degree of hesitation, use the following expression:

$$\pi_P(j_i) = 1 - \mu_P(j_i) - \nu_P(j_i); \forall \, j_i \in J. \tag{9}$$

It is obvious that $\pi_P(j_i) \in [0,1]$ When $\pi_P(j_i) = 0$, the AInF-set degenerates into an ordinary fuzzy set. The greatest AInF-set is one in which each element's values for the membership and non-membership functions are the same. In most AInF-sets, each element is referred to as an overlap member.

Note: In this work, we will refer to AInFS(J) as a collection of all AInF-sets defined on J.

Definition 3. For two AInF-set P and Q in J, the relations listed below can be described as follows:

$$P = \{ < j_i, \mu_P(j_i), \nu_P(j_i) > : j_i \in J\},$$
$$Q = \{ < j_i, \mu_Q(j_i), \nu_Q(j_i) > : j_i \in Q\},$$

then these are the basic operations on an AInF-set:

$$\begin{aligned} &P \cap Q = \{ < j_i, min\big(\mu_P(j_i), \mu_Q(j_i)\big), max\big(\nu_P(j_i), \nu_Q(j_i)\big) > : j_i \in J\};\\ &P \cup Q = \{ < j_i, max\big(\mu_P(j_i), \mu_Q(j_i)\big), min\big(\nu_P(j_i), \nu_Q(j_i)\big) > : j_i \in J\};\\ &P^c = \{ < j_i, \nu_P(j_i), \mu_P(j_i) > : j_i \in J\};\\ &P \subseteq Q \Leftrightarrow \begin{cases} \mu_P(j_i) \le \mu_Q(j_i) \text{ and } \nu_P(j_i) \ge \nu_Q(j_i) \text{ if } \mu_P(j_i) \le \nu_Q(j_i), \\ \mu_P(j_i) \ge \mu_Q(j_i) \text{ and } \nu_P(j_i) \le \nu_Q(j_i) \text{ if } \mu_P(j_i) \ge \nu_Q(j_i); \end{cases}\\ &P = Q \Leftrightarrow P \subseteq Q \text{ and } Q \subseteq P. \end{aligned} \tag{10}$$

Definition 4. ([87]) For a function N : AInFS(J) → [0,1] to be defined as an AInF-information measure, it has to satisfy all four of the following axioms:

(N1) $N(P) = 0$ iff $\mu_P(j_i) = 0, \nu_P(j_i) = 1$ or $\mu_P(j_h) = 1, \nu_P(j_i) = 0\ \forall\, j_i \in J$, i.e., P is a least AInFset.
(N2) $N(P) = 1$ iff $\mu_P(j_i) = \nu_P(j_i) \forall j_i \in J$, i.e., P is a most AInf – set.
(N3) $N(P) \le N(Q)$ iff $P \subseteq Q$.
(N4) $N(P) = N(P^c)$, where P^c is the complement of P.

The fuzzy entropy establishes the fuzzyness of a fuzzy collection. A knowledge measure also establishes the overall amount of knowledge. [88] claim that these two ideas complement one another.

Definition 5. ([88] In order for a function *M : AInFS(J)* → [0,1] to be classified as an AInF-knowledge measure, it has to meet four specific axioms:

(M1) $M(P) = 1$ iff $\mu_P(j_i) = 0, \nu_P(j_i) = 1$ or $\mu_P(j_i) = 1, \nu_P(j_i) = 0\ \forall\, j_i \in J$, i.e., P is a least AIF – set.
(M2) $M(P) = 0$ iff $\mu_P(j_i) = \nu_P(j_i)\ \forall\, j_i \in J$, i.e., P is a most AIF – set.
(M3) $M(P) \ge M(Q)$ iff $P \subseteq Q$.
(M4) $M(P) = M(P^c)$, where $P^c$is the complement of P.

Definition 6. ([89,90]) Let P, Q, R ∈ *AInFS(J)*. For a mapping F_n: *AInFS(J)*× *AInFS(J)* → [0,1] to be considered an AInF-similarity measure, it must satisfy the following four axioms:

(F1) $0 \le F_n(P, Q) \le 1$.
(F2) $F_n(P, Q) = F_n(Q, P)$.
(F3) $F_n(P, Q) = 1 \Leftrightarrow P = Q$.
(F4) $P \subseteq Q \subseteq R$, then $F_n(P, Q) \ge F_n(P, R)$ and $F_n(Q, R) \ge F_n(P, R)$.

Definition 7. ([91]) Let P, Q, R ∈ *AInFS(J)*. For a mapping G_n : *AInFS(J)* ×*AInFS(J)* → [0,1] to be considered an AInF-dissimilarity measure, it must satisfy the following four axioms:

(G1) $0 \le G_n(P, Q) \le 1$.
(G2) $G_n(P, Q) = G_n(Q, P)$.
(G3) $G_n(P, Q) = 0 \Leftrightarrow P = Q$.
(G4) $P \subseteq Q \subseteq R$, then $G_n(P, Q) \le G_n(P, R)$ and $G_n(Q, R) \le G_n(P, R)$.

Definition 8. Let P, Q ∈ *AInFS(J)*. If a mapping H_n : *AInFS(J)* × *AInFS(J)* → [0,1] satisfies all four of the following axioms, it is said to be accuracy measure in P w.r.t. Q:

(H1) $H_n(P, Q) \in [0,1]$.

(H2) $H_n(P,Q) = 0 \Leftrightarrow \mu_P(j_i) = \mu_Q(j_i)$.

(H3) $H_n(P,Q) = 1$ if $\mu_P(j_i) = 0 = \mu_Q(j_i), \nu_P(j_i) = 1 = \nu_Q(j_i)$ or $\mu_P(j_i) = 1 = \mu_Q(j_i), \nu_P(j_i) = 0 = \nu_Q(j_i)\ \forall\ j_i \in J$, i.e., Both P and Q are equal and least AInF − set.

(H4) $H_n(P,Q) = M(P)$ if $P = Q$, where $M(P)$is knowledge measure.

[92] outlined a procedure that converts AInF-sets into fuzzy sets, this procedure is covered in full here.

Definition 9. ([92]) Let P ∈ *AInFS(J)*, then the membership function $\mu_P(j_i)$ corresponding to fuzzy set $\bar{P}$ is provided below

$$\begin{aligned}\mu_{\bar{P}}(j_i) &= \mu_P(j_i) + \frac{\pi_P(j_i)}{2};\\ &= \frac{\mu_P(j_i) + 1 - \nu_P(j_i)}{2}, \forall j_i \in J.\end{aligned} \tag{11}$$

[49] defined fuzzy entropy for a fuzzy set $\bar{P}$ corresponding to Eq.(2) as

$$C(\bar{P}) = -\frac{1}{w}\sum_{i=1}^{h}\left[\mu_{\bar{P}}(j_i)log\left(\mu_{\bar{P}}(j_i)\right) + \left(1-\mu_{\bar{P}}(j_i)\right)log\left(1-\mu_{\bar{P}}(j_i)\right)\right]. \tag{12}$$

A research by [93] providing a measure of fuzzy entropy was undertaken on information measures on fuzzy sets. In accordance with Eq. (3) , they advise doing the following action:

$$C_p(\bar{P}) = -\frac{1}{(1-p)}\sum_{i=1}^{h} log\left[\mu_{\bar{P}}^{p}(j_i) + \left(1-\mu_{\bar{P}}(j_i)\right)^{p}\right], p \neq 1, p > 0. \tag{13}$$

We suggested an AInF-knowledge measure in the next section.

3. A NEW EXPONENTIAL KNOWLEDGE MEASURE FOR AInF-Sets

We build an improved AInF-knowledge measure (AKM) in this part using the fuzzy entropy measure from [85,86] as follows.

$$G_H^I(P) = \frac{1}{h(1-e^{0.5})}\sum_{i=1}^{h}\left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)} + \left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)} - e^{0.5}\right] \tag{14}$$

assuming a $P \in$ *AInFS(J)*. All the information that may be captured by the suggested AInF-knowledge measure is displayed in Figure 1. Now, we test the proposed measure $G_H^I(P)$ for its validity.

Theorem 1. For a finite set $J(\neq \phi)$, take $P = \{< j_i, \mu_P(j_i), \nu_P(j_i) >: j_i \in J\}$ and $Q = \{< j_i, \mu_Q(j_i), \nu_Q(j_i) >: j_i \in J\}$ as an element of *AInFS(J)* . Consider the function G_H^I : *AInFS(J)* → [0,1] defined by Eq. (14) . For the function G_H^I to be called a knowledge measure for AInF-sets, the following axioms (K1)-(K4) must be true:

$(K1)\ G_H^I(P) = 1\ iff\ \mu_P(j_i) = 0, \nu_P(j_i) = 1\ or\ \mu_P(j_i) = 1, \nu_P(j_i) = 0\ \forall\ j_i \in J, i.e., P\ is\ a\ least\ AInF - set.$

$(K2)\ G_H^I(P) = 0\ iff\ \mu_P(j_i) = \nu_P(j_i)\ \forall\ j_i \in J, i.e., P\ is\ a\ most\ AInF - set.$

$(K3)\ G_H^I(P) \geq G_H^I(P)\ iff\ P \subseteq Q.$

$(K4)\ G_H^I(P) = G_H^I(P^c), where\ P^c\ is\ the\ complement\ of AInF - setG.$

(**K1**). First we suppose that $G_H^I(P) = 1$

$$\Leftrightarrow \frac{1}{h(1-e^{0.5})}\sum_{i=1}^{h}\left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)}\right.$$
$$\left.+\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)} - e^{0.5}\right] = 1,$$
$$\Leftrightarrow \left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)}\right.$$
$$\left.+\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)}\right] = 1, \forall j_i \in J,$$

$$\Leftrightarrow \mu_P(j_i) = 0, \nu_P(j_i) = 1 \text{ or } \mu_P(j_i) = 1, \nu_P(j_i) = 0 \ \forall\ j_i \in J$$

Thus, $G_H^I(P) = 1$ for a least AInF-set P.

(**K2**). Let us take $G_H^I(P) = 0$. Then, from Eq. (14), we have

$$\frac{1}{h(1-e^{0.5})}\sum_{i=1}^{h}\left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)}\right.$$
$$\left.+\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)} - e^{0.5}\right] = 0,$$

which gives

$$\left[\left(\mu_P(j_i)+1-\nu_P(j_i)\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)} + \left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)}\right] = e^{0.5}, \forall j_i \in J.$$

Thus, we get $\mu_P(j_i) = \nu_P(j_i)\ \forall\ j_i \in J$.
Conversely, Let $\mu_P(j_i) = \nu_P(j_i)\ \forall\ j_i \in J$, then Eq. (14) implies $G_H^I(P) = 0$.
Thus, $G_H^I(P)$ =0 $\Leftrightarrow$ P is the most AInF-set.

(**K3**). First, consider a function

$$m(r,s) = \left[\left(\frac{r+1-s}{2}\right)e^{\left(\frac{s+1-r}{2}\right)} + \left(\frac{s+1-r}{2}\right)e^{\left(\frac{r+1-s}{2}\right)} - e^{0.5}\right] \tag{15}$$

is a function that increases with respect to s and decrease with respect to r, where r,s $\in$ [0,1]. Differentiate function m partially w.r.t. r, we obtain

$$\frac{\partial m(r,s)}{\partial r} = \left[\frac{1}{2}e^{\left(\frac{s+1-r}{2}\right)} - \frac{1}{2}\left(\frac{r+1-s}{2}\right)e^{\left(\frac{s+1-r}{2}\right)} - \frac{1}{2}e^{\left(\frac{r+1-s}{2}\right)} + \frac{1}{2}\left(\frac{s+1-r}{2}\right)e^{\left(\frac{r+1-s}{2}\right)}\right] \tag{16}$$

It is now possible to find critical points of r by entering

$$\frac{\partial m(r,s)}{\partial r} = 0;$$

which gives r = s.
Here, two cases are mentioned below:

$$\frac{\partial m(r,s)}{\partial r} = \begin{cases} positive\ if\ r \geq s \\ negative\ if\ r \leq s \end{cases} \tag{17}$$

i.e., function m is lowering function for $r \leq s$ and raising function for $r \geq s$.
Likewise, we possess

$$\frac{\partial m(r,s)}{\partial r}=\begin{cases} negative\ if\ r\geq s \\ positive\ \ if\ r\leq s \end{cases} \tag{18}$$

i.e., function m is lowering function for $r\leq\ s$ and raising function for $r\geq\ s$.
Now, take P,Q ∈ AInFS(J) s.t. P ⊆ Q. Let J_1 and J_2 are two partitions of J s.t. $J=J_1\ \cup\ J_2$ and

$$\begin{cases} \mu_P(j_i)\leq\mu_Q(j_i)\leq\upsilon_P(j_i)\leq\upsilon_P(j_i)\ \forall\ j_i\in J_1, \\ \mu_P(j_i)\geq\mu_Q(j_i)\leq\upsilon_P(j_i)\geq\upsilon_P(j_i)\ \forall\ j_i\in J_2. \end{cases}$$

Thus function m is monotonic and because of Eq.(14), it is thus simple to demonstrate that $G_H^I(P)\geq G_H^I(Q)$.
This validates axiom (K3).

(**K4**). It is simple to observe that

$$P^c=\{<j_i,\upsilon_P(j_i),\ \mu_P(j_i)>:j_i\in J\},$$

i.e., $\mu_{P^c}(j_i)=\upsilon_P(j_i)$ and $\mu_P(j_i)=\upsilon_{P^c}(j_i)\ \forall\ j_i\in J$. Thus, from Eq. (14), we get $G_H^I(P)=G_H^I(P^c)$. This validates axiom (K4).
As a result, $G_H^I(P)$ is an accurate AInF-knowledge measure.
Following that, we will go over the few properties of the suggested knowledge measure for AInF-set.

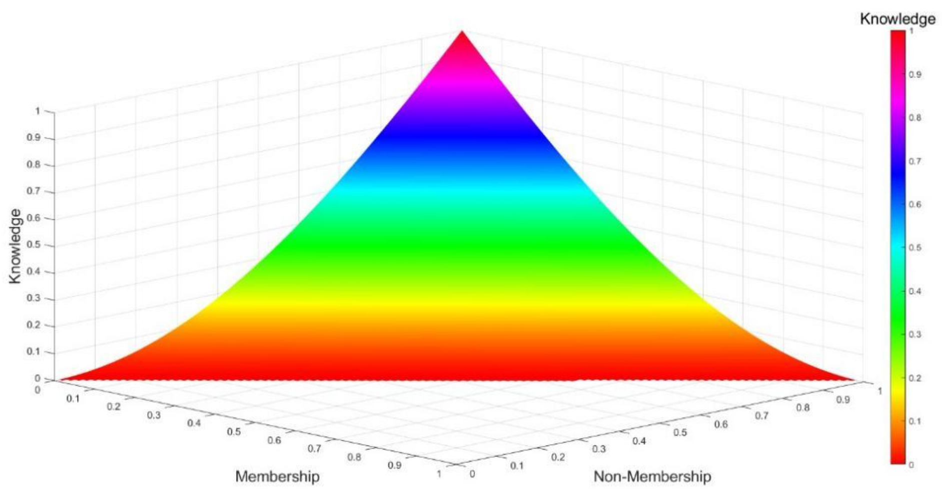

Figure 1. Diagrammatical representation of proposed AInF-knowledge measure.

3.1. Some Properties

Theorem 2. The measure $G_H^I(P)$ satisfied the following properties:
(1) $G_H^I(P)=G_H^I(P^c)$
(2) $G_H^I(P\cup Q)+G_H^I(P\cap Q)=G_H^I(P)+G_H^I(Q)$ for any two arbitrary AInF-sets P, Q.
(3) For a least AInF-set, measure $G_H^I(P)$ attains its maximum value, and for most AInF-set, measure $G_H^I(P)$ attains its minimum value.

Proof

(1). From the axiom (K4), the proof is obvious.

(2). Let P, Q ∈ *AIFS(J)*. Break J into two subsets that are specified as follows:

$$J_1=\{j_i\in J|P\subseteq Q\}, J_2=\{j_i\in J|P\subseteq Q\}, \tag{19}$$

i.e.,

$$\begin{cases}\mu_P(j_i) \le \mu_Q(j_i) \text{ and } v_P(j_i) \ge v_Q(j_i)\ \forall\, j_i \in J_1,\\ \mu_P(j_i) \ge \mu_Q(j_i) \text{ and } v_P(j_i) \le v_q(j_i)\ \forall\, j_i \in J_2.\end{cases}$$

where $\mu_P(j_i)$ and $\mu_q(j_i)$ are the membership functions and $v_P(j_i)$ and $v_q(j_i)$ are the non-membership functions for AInF-set P and Q, respectively. Now, $\forall\, j_i \in J$, we have

$$G_H^I(P\cup Q)+G_H^I(P\cap Q) = \frac{1}{h(1-e^{0.5})}\sum_{i=1}^{h}\left[\begin{array}{c}\left(\dfrac{\mu_{PuQ}(j_i)+1-v_{PuQ}(j_i)}{2}\right)e^{\left(\frac{v_{PuQ}(j_i)+1-\mu_{PuQ}(j_i)}{2}\right)}\\ +\left(\dfrac{v_{P\cup Q}(j_i)+1-\mu_{P\cup Q}(j_i)}{2}\right)e^{\left(\frac{\mu_{P\cup Q}(j_i)+1-v_{P\cup Q}(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

$$+\frac{1}{h(1-e^{0.5})}\sum_{i=1}^{h}\left[\begin{array}{c}\left(\dfrac{\mu_{PrQ}(j_i)+1-v_{PnQ}(j_i)}{2}\right)e^{\left(\frac{v_{PnQ}(j_i)+1-\mu_{PnQ}(j_i)}{2}\right)}\\ +\left(\dfrac{v_{PnQ}(j_i)+1-\mu_{PnQ}(j_i)}{2}\right)e^{\left(\frac{\mu_{PnQ}(j_i)+1-v_{PnQ}(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

which gives

$$G_H^I(P\cup Q)+G_H^I(P\cap Q) = \frac{1}{h(1-e^{0.5})}\sum_{j_1}\left[\begin{array}{c}\left(\dfrac{\mu_Q(j_i)+1-v_Q(j_i)}{2}\right)e^{\left(\frac{v_Q(j_i)+1-\mu_Q(j_i)}{2}\right)}\\ +\left(\dfrac{v_Q(j_i)+1-\mu_Q(j_i)}{2}\right)e^{\left(\frac{\mu_Q(j_i)+1-v_Q(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

$$+\frac{1}{h(1-e^{0.5})}\sum_{j_2}\left[\begin{array}{c}\left(\dfrac{\mu_P(j_i)+1-v_P(j_i)}{2}\right)e^{\left(\frac{v_P(j_i)+1-\mu_P(j_i)}{2}\right)}\\ +\left(\dfrac{v_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-v_P(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

$$\frac{1}{h(1-e^{0.5})}\sum_{j_2}\left[\begin{array}{c}\left(\dfrac{\mu_Q(j_i)+1-v_Q(j_i)}{2}\right)e^{\left(\frac{v_Q(j_i)+1-\mu_Q(j_i)}{2}\right)}\\ +\left(\dfrac{v_Q(j_i)+1-\mu_Q(j_i)}{2}\right)e^{\left(\frac{\mu_Q(j_i)+1-v_Q(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

$$+\frac{1}{h(1-e^{0.5})}\sum_{j_i}\left[\begin{array}{c}\left(\dfrac{\mu_P(j_i)+1-v_P(j_i)}{2}\right)e^{\left(\frac{v_P(j_i)+1-\mu_P(j_i)}{2}\right)}\\ +\left(\dfrac{v_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-v_P(j_i)}{2}\right)}-e^{0.5}\end{array}\right]$$

Further solving gives us

$$G_H^I(P\cup Q)+G_H^I(P\cap Q) = G_H^I(P)+G_H^I(Q). \tag{20}$$

(3). The proof follows directly from the axioms (K1) and (K2) .

3.2. Comparison

We now examine how well the suggested measure performs in the AInF-context in comparison to the previously estab- lished measures. Benefits of the proposed measure are assessed in this comparison. We investigate these advantages with regard to the evaluation of ambiguity content of AInF-sets, the estimation of characteristics weights inside MCDM issues, and the manipulation of structured linguistic variables. Some measures in the AInF-context are as follows:

$$N_{LZ}(P) = 1-\frac{1}{h}\sum_{i=1}^{h}|\mu_P(j_i)-v_P(j_i)|\,;[94] \tag{21}$$

$$N_{BB}(P) = \frac{1}{h}\sum_{i=1}^{h} |1 - \mu_P(j_i) - v_P(j_i)| \,; [51]. \tag{22}$$

$$I_{KS}(P) = \frac{1}{h}\sum_{i=1}^{h} \frac{min\big(\mu_P(j_i), v_P(j_i)\big) + \pi_P(j_i)}{max\big(\mu_P(j_i), v_P(j_i)\big) + \pi_P(j_i)}; [87] \tag{23}$$

$$N_{YH}(P) = \frac{1}{h}\sum_{i=1}^{h} \left(1 - \mu_P^2(j_i) - v_P^2(j_i) - \pi_P^2(j_i)\right); [95] \tag{24}$$

$$N_{JZ}(P) = \frac{1}{h}\sum_{i=1}^{h} \frac{min\big(\mu_P(j_i), v_P(j_i)\big)}{max\big(\mu_P(j_i), v_P(j_i)\big)}; [96]. \tag{25}$$

$$N_L^q(P) = 1 - \frac{1}{2h}\sum_{i=1}^{h} \left(|\mu_P(j_i) - v_P(j_i)|^q + |\mu_P(j_i) - v_P(j_i)|^{3q}\right), q > 0; [97] \tag{26}$$

$$N_J^y(P) = \frac{y}{h(1-y)}\sum_{i=1}^{h} \left(1 - \left(\mu_P^y(j_i) + v_P^y(j_i) + \pi_P^y(j_i)\right)^{\frac{1}{y}}\right); [98]. \tag{27}$$

$$M_S(P) = 1 - \frac{1}{2h}\sum_{i=1}^{h} \left[\frac{min\big(\mu_P(j_i), v_P(j_i)\big) + \pi_P(j_i)}{max\big(\mu_P(j_i), v_P(j_i)\big) + \pi_P(j_i)} + \pi_P(j_i)\right]; [58] \tag{28}$$

$$M_N(P) = \frac{1}{h\sqrt{2}}\sum_{i=1}^{h} \sqrt{\mu_P^2(j_i) + v_P^2(j_i) + \big(\mu_P(j_i) + v_P(j_i)\big)^2}\,; [67]. \tag{29}$$

$$M_U(P) = 1 - \frac{1}{2h}\sum_{i=1}^{h} \left(1 - |\mu_P(j_i) - v_P(j_i)|\right)\left(1 + \pi_P(j_i)\right); [59]. \tag{30}$$

$$\begin{aligned} G_H^I(P) = \frac{1}{h\left(1 - e^{0.5}\right)}\sum_{i=1}^{h} \Bigg[&\left(\frac{\mu_P(j_i) + 1 - v_P(j_i)}{2}\right) e^{\left(\frac{v_P(j_i)+1-\mu_P(j_i)}{2}\right)} \\ &+ \left(\frac{v_P(j_i) + 1 - \mu_P(j_i)}{2}\right) e^{\left(\frac{\mu_P(j_i)+1-v_P(j_i)}{2}\right)} - e^{0.5}\Bigg] \end{aligned} \tag{31}$$

(Proposed One)

3.2.1. Structured Linguistic Contrast

Linguistic variables can be replaced with AInF-sets. Moreover, operations on these variables are defined using linguistic hedges. The most popular linguistic hedges are "VERY," "FEW," and "SLIGHTLY". They reflect linguistic variations. The proposed measure and a few additional measurements are contrasted below:

Let $J(\neq \phi)$ be finite set and take $P \in$ *AInFS(J)* be s.t. $P = \{< j_i, \mu_P(j_i), v_P(j_i) >: j_i \in J\}$. We consider this AInF-set as "Large". For $t > 0$, [99] established the modifier for an AInF-set P as follows

$$P^t = \{< j_i, \left(\mu_P(j_i)\right)^t, 1 - (1 - v_P)^t(j_i) >: j_i \in J\}. \tag{32}$$

The following defines the Dilatation and Concentration of an AInF-set P:

$$\begin{aligned} P^{CON} &= P^2, \\ P^{DIL} &= P^{0.5}. \end{aligned} \tag{33}$$

The modifiers provided above can be used to determine the Concentration and Dilatation for an AInF-set. We utilise the following acronyms for clarity: Z is used for Large, ZVV is used for Very Very Large, ZQV is used for Quite Very Large, ZV is used for Very Large, and ZML is used for More large. For an AInF-set P, we may define the hedges by

$$\begin{cases} Z^{ML} \ \ stands\ for\ P^{0.5} \\ Z \quad stands\ for\ P \\ Z^{V} \ \ stands\ for\ P^{2} \\ Z^{QV}\ stands\ for\ P^{3} \\ Z^{VV}\ \ stands\ for\ P^{4} \end{cases} \tag{34}$$

According to published research, the degree of ambiguity rises and the degree of knowledge falls as one moves from $P^{0.5}$ to P^4. The following conditions must be met for any information measure N to be performed appropriately:

$$N(Z^{VV}) < N(Z^{QV}) < N(Z^{V}) < N(Z) < N(Z^{ML}); \tag{35}$$

where $N(P)$ is an information measure defined for AInF-set P. On the other hand, the following order has to be met in order for any knowledge measure M to be considered validly performed:

$$M(Z^{VV}) > M(Z^{QV}) > M(Z^{V}) > M(Z) > M(Z^{ML}); \tag{36}$$

where $M(P)$ is a knowledge measure defined for AInF-set P.

In order to compare the suggested measure with an alternative measure, we now use the following example:

Example 1. Let $J = \{ji,\ 1 \le i \le 5\}$ and take a AInF-set P defined on the set J as follows:

$$P = \{(j_1, 0.105, 0.809), (j_2, 0.297, 0.492), (j_3, 0.509, 0.482), (j_4, 0.906, 0.005), (j_5, 0.997, 0.001)\}. \tag{37}$$

We denote the AInF-set P as "Large" on J and specified the linguistic variables according to Eq.(34). The values of the modifiers are determined using Eq.(32) and are provided by

$$\begin{aligned} P^{0.5} &= \{(j_1, 0.3240, 0.5630), (j_2, 0.5450, 0.2873), (j_3, 0.7134, 0.2803), (j_4, 0.9518, 0.0025), (j_5, 0.9985, 0.0005)\}; \\ P &= \{(j_1, 0.1050, 0.8090), (j_2, 0.2970, 0.4920), (j_3, 0.5090, 0.4820), (j_4, 0.9060, 0.0050), (j_5, 0.9970, 0.0010)\}; \\ P^2 &= \{(j_1, 0.0110, 0.9635), (j_2, 0.0882, 0.7419), (j_3, 0.2591, 0.7317), (j_4, 0.8208, 0.0100), (j_5, 0.9940, 0.0020)\}; \\ P &= \{(j_1, 0.0012, 0.9930), (j_2, 0.0262, 0.8689), (j_3, 0.1319, 0.8610), (j_4, 0.7437, 0.0149), (j_5, 0.9910, 0.0030)\}; \\ p &= \{(j_1, 0.0001, 0.9987), (j_2, 0.0078, 0.9334), (j_3, 0.0671, 0.9280), (j_4, 0.6738, 0.0199), (j_5, 0.9881, 0.0040)\}. \end{aligned} \tag{38}$$

We now use the suggested measure together with a few known measures to find the information and knowledge transferred by these sets. Table 1 presents a comparison and illustration between the values obtained from the existing measures and the proposed AInF-knowledge measure

Table 1. Comparison of the suggested measure with known previous measures

Measures →AInF-Set↓	$N_{LZ}(P)$	$N_{BB}(P)$	$N_{KS}(P)$	$N_{YH}(P)$	$N_{JZ}(P)$	$N_L^q(P)$	$N_J^y(P)$	$M_S(P)$	$M_N(P)$	$M_U(P)$	$G_H^l(P)$
$P^{0.5}$	0.4246	0.0667	0.3466	0.3330	0.2997	0.6429	0.3038	0.7933	0.8698	0.7661	0.4384
P	0.4354	0.0794	0.3963	0.3276	0.3374	0.6526	0.3042	0.7622	0.8642	0.7610	0.4502
P^2	0.2237	0.0755	0.1738	0.2381	0.0997	0.5491	0.1868	0.8753	0.8939	0.8785	0.6331
P^3	0.1439	0.0730	0.1142	0.1789	0.0415	0.4530	0.1354	0.9064	0.9107	0.9196	0.6570
P^4	0.1154	0.0759	0.0981	0.1475	0.0229	0.3708	0.1189	0.9130	0.9147	0.9312	0.6702

We take $q = 3$ for $N_L^q(G)$ and $y = 5$ for $N_J^y(G)$.

The subsequent observations are predicated upon Table 1.

$$
\begin{aligned}
&N_{LZ}(Z^{VV}) < N_{LZ}(Z^{QV}) < N_{LZ}(Z^{V}) < N_{LZ}(Z) > N_{LZ}(Z^{ML});\\
&N_{BB}(Z^{VV}) > N_{BB}(Z^{QV}) < N_{BB}(Z^{V}) < N_{BB}(Z) > N_{BB}(Z^{ML})\\
&N_{KS}(Z^{VV}) < N_{KS}(Z^{QV}) < N_{KS}(Z^{V}) < N_{KS}(Z) > N_{KS}(Z^{ML});\\
&N_{YH}(Z^{VV}) < N_{YH}(Z^{QV}) < N_{YH}(Z^{V}) < N_{YH}(Z) < N_{YH}(Z^{ML});\\
&N_{JZ}(Z^{VV}) < N_{JZ}(Z^{QV}) < N_{JZ}(Z^{V}) < N_{JZ}(Z) > N_{JZ}(Z^{ML});\\
&N_L^3(Z^{VV}) < N_L^3(Z^{QV}) < N_L^3(Z^{V}) < N_Z^3(L) > N_L^3(Z^{ML});\\
&N_J^5(Z^{VV}) < N_J^5(Z^{QV}) < N_J^5(Z^{V}) < N_J^5(Z) > N_J^5(Z^{ML});\\
&M_S(Z^{VV}) > M_S(Z^{QV}) > M_S(Z^{V}) > M_S(Z) < M_S(Z^{ML});\\
&M_N(Z^{VV}) > M_N(Z^{QV}) > M_N(Z^{V}) > M_S(Z) < M_N(Z^{ML})\\
&M_U(Z^{VV}) > M_U(Z^{QV}) > M_U(Z^{V}) > M_U(Z) < M_U(Z^{ML});\\
&G_H^I(Z^{VV}) > G_H^I(Z^{QV}) > G_H^I(Z^{V}) > G_H^I(Z) > G_H^I(Z^{ML}).
\end{aligned} \tag{39}
$$

We now found that, except for $N_{YH}(P)$ and $G_H^I(P)$, no knowledge and information measure matches the order given by Eqs. (35) and (36). It means that things aren't going well for them. We thus just need to compare knowledge measure $G_H^I(P)$ and information measure $N_{YH}(P)$.

We use a different AInF-set generated by

$$P = \{(j_1, 0.110, 0.798), (j_2, 0.280, 0.502), (j_3, 0.475, 0.423), (j_4, 0.920, 0.019), (j_5, 0.981, 0.005)\}. \tag{40}$$

Table 2 presents a comparison and illustration between the values obtained from the existing measure and the proposed AInF-knowledge measure.

Table 2. Comparison of the suggested measure with known previous measures

AInF-Set→Measures↓	$P^{0.5}$	P	P^2	P^3	P^4
$N_{YH}(P)$	0.3471	0.3473	0.2626	0.2072	0.1798
$G_H^I(P)$	0.4496	0.4906	0.64	0.7466	0.7821

$$
\begin{aligned}
&N_{YH}(Z^{VV}) < N_{YH}(Z^{QV}) < N_{YH}(Z^{V}) < N_{YH}(Z) > N_{YH}(Z^{ML});\\
&G_H^I(Z^{VV}) > G_H^I(Z^{QV}) > G_H^I(Z^{V}) > G_H^I(Z) > G_H^I(Z^{ML}).
\end{aligned} \tag{41}
$$

Here, we can see that the information measure deviates from the order specified by Eq. (35). The suggested knowledge measure is arranged appropriately, though. Consequently, the proposed measure' s performance is outstanding.

3.2.2. Uncertainty Calculation

The amount of ambiguity associated with two distinct AInF-sets varies. However, given two distinct AInF-sets, the uncertainty offered by certain measure is the same. Here, we use the suggested measure together with existing measures from the past to compute the overall amount of uncertainty associated with two distinct AInF-sets. Consider the following example for this:

Example 2. Define a set $J = \{j_1, j_2, j_3, j_4\}$ and take $J_1, J_2, J_3, J_4 \in AInFS(J)$ as follows:

$$
\begin{aligned}
P_1 &= \{(j_1, 0.423, 0.529), (j_2, 0.219, 0.421), (j_3, 0.231, 0.480), (j_4, 0.421, 0.368)\};\\
P_2 &= \{(j_1, 0.320, 0.480), (j_2, 0.410, 0.390), (j_3, 0.480, 0.320), (j_4, 0.319, 0.481)\};\\
P_3 &= \{(j_1, 0.623, 0.077), (j_2, 0.619, 0.080), (j_3, 0.613, 0.065), (j_4, 0.725, 0.002)\};\\
P_4 &= \{(j_1, 0.423, 0.019), (j_2, 0.214, 0.523), (j_3, 0.329, 0.112), (j_4, 0.298, 0.397)\}
\end{aligned} \tag{42}
$$

We use the suggested knowledge measure and prior existing measures to quantify the amount of uncertainty associated with particular AInF-sets. Table 3 displays all of the findings. From Table 3 it is observed that the

Table 3. Uncertainty related to different AInF-sets

AInF-Set→Measures↓	P_1	P_2	P_3	P_4

$M_S(P)$	**0.4927**	**0.4927**	0.6615	0.4382
$M_N(P)$	**0.6754**	0.6963	**0.6754**	0.5226
$M_U(P)$	0.4829	**0.4753**	0.7325	**0.4753**
$G_H^I(P)$	0.0282	0.0186	0.2311	0.0758

uncertainty calculated by some previously known measures is showing variation for different AInF-sets. However, the outcomes of the suggested measure are optimistic. Therefore, a new measure is required for AInF-sets.

3.2.3. Assessing Criteria Weights

In any MCDM, MADM, or MAGDM problem, criteria weights are extremely important. A slight change in the weights of the criteria can have a significant impact on the outcomes of decision-making problems. This is demonstrated by the following example.

Example 3. Consider a matrix of decisions C with the set of choices $\{A_1, A_2, A_3, A_4\}$ and set of attributes $\{B_1, B_2, B_3, B_4\}$ developed in an intuitionistic fuzzy environment.

$$C = \begin{bmatrix} <0.623,0.077> & <0.320,0.480> & <0.423,0.019> & <0.423,0.529> \\ <0.619,0.080> & <0.410,0.390> & <0.214,0.523> & <0.219,0.421> \\ <0.613,0.065> & <0.480,0.320> & <0.329,0.112> & <0.231,0.480> \\ <0.725,0.002> & <0.480,0.320> & <0.329,0.112> & <0.231,0.480> \end{bmatrix}$$

One of the two techniques listed below is used to determine the attribute weights:

(1) Entropy-based approach: The weights assigned to various qualities may be found using the following formula:

$$\omega_j = \frac{1 - N(B_j)}{\sum_{j=1}^{t} s - N(B_j)}, j = 1,2,3\ldots s; \tag{43}$$

where N is the information measure for AInF-set.

(2) Knowledge-based approach: The weights assigned to different traits may be found using the following formula:

$$\omega_j = \frac{M(B_j)}{\sum_{j=1}^{t} M\left(B_j\right)}, j = 1,2,3\ldots s; \tag{44}$$

where M is the knowledge measure for AInF-set.

Table 4. Calculation of criteria weights

Criteria weights→Measures↓	ω_1	ω_2	ω_3	ω_4
$M_S(P)$	0.3173	**0.2363**	0.2101	**0.2363**
$M_N(P)$	**0.2628**	0.2710	0.2034	**0.2628**
$M_U(P)$	0.3382	**0.2194**	0.2194	0.2229
$G_H^I(P)$	0.6533	0.0525	0.2143	0.07997

It is clear from Table 4 that the previously established measures provide criteria weights that are equivalent to an alternative. However, the suggested method produces adequate criterion weight results for any problem requiring decision-making. This implies that a measure for AInF-sets is required.

The accuracy measure for AInF-sets that we deduced from the suggested measure will be discussed in the next section.

4. Deduction

4.1. Accuracy Measure for AInF-sets

The quantity of AInF-knowledge and accuracy are equal. When we want to know how accurate one AInF-set Q is in contrast to another AInF-set P, we employ the concept of the AInF-accuracy measure. Following the development of the concept of an AIF-set inaccuracy measure generated from fuzzy arrays, [100] produced the intuitionistic fuzzy inaccuracy measure that follows:

$$I(P,Q) = -\frac{1}{h}\sum_{i=1}^{h}\left[\mu_P log\left(\frac{\mu_P+\mu_Q}{2}\right)+\nu_P log\left(\frac{\nu_P+\nu_Q}{2}\right)+\pi_P log\left(\frac{\pi_P+\pi_Q}{2}\right)-\pi_P log\pi_P-(1-\pi_P)log(1-\pi_P)-\pi_P\right]; \quad (45)$$

where P,Q ∈ *AInFS(J)*.

In the framework of AInF, we develop an accuracy measure based on the proposed measure. Let P, Q ∈ *AInFS(J)* for a finite set $J(\neq \phi)$. We derive an AInF-accuracy measure $G^I_{accy}(P, Q)$ of AInF-set Q w.r.t AInF-set P by

$$G^I_{accy}(P,Q) = \frac{1}{2h(1-e^{0.5})}\sum_{i=1}^{h}\left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)}+\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)}-e^{0.5}\right]$$
$$+\frac{1}{2h(1-e^{0.5})}\sum_{i=1}^{h}\left[\begin{array}{l}\sqrt{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)\times\left(\frac{\mu_Q(j_i)+1-\nu_Q(j_i)}{2}\right)}e^{\sqrt{\left(\frac{1+\nu_G(z_i)-\mu_G(z_i)}{2}\right)\times\left(\frac{1+\nu_H(z_i)-\mu_H(z_i)}{2}\right)}}\\+\sqrt{\left(\frac{1+\nu_G(z_i)-\mu_G(z_i)}{2}\right)\times\left(\frac{1+\nu_H(z_i)-\mu_H(z_i)}{2}\right)}e^{\sqrt{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)\times\left(\frac{\mu_Q(j_i)+1-\nu_Q(j_i)}{2}\right)}}-e^{0.5}\end{array}\right] \quad (46)$$

Now, we test the proposed accuracy measure G^I_{accy} for its validity.

Theorem 3. Let *P, Q* ∈ *AInFS(J)* , where $J(\neq \phi)$ is a finite set. Consider the function G^I_{accy}: *AInFS(J)* × *AInFS(J)* → [0, 1] defined by Eq. (46). For the function G^I_{accy} to be called an accuracy measure for AInF-sets P and Q, the following axioms (E1)-(E4) must be true:

$(E1) G^I_{acey}(P,Q) = 1\ if\ \mu_P(j_i) = \mu_Q(j_i) = 0, \nu_P(j_i) = \nu_Q(j_i) = 1\ or\ \mu_P(j_i) = \mu_Q(j_i) = 1, \nu_P(j_i) = \nu_Q(j_i) = 0$ $\forall j_i \in J, i.e., P\ and\ Q\ both\ are\ equal\ least\ AInF-set.$
$(E2) G^I_{acey}(P,Q) = 0 \Leftrightarrow \mu_P(j_i) = \nu_P(j_i).$
$(E3) G^I_{acey}(P,Q) \in [0,1].$
$(E_4) G^I_{acey}(P,Q) = G^I_H(P) \oint G = H. where\ G^I_H(P)\ is\ the\ proposed\ AInF-knowledge\ measure.$

Proof

(E1). Assume that P and Q are the two least comparable AInF-sets. It implies that $\mu_P(j_i) = \mu_Q(j_i) = 0, \nu_P(j_i) = \nu_Q(j_i) = 1\ or\ \mu_P(j_i) = \mu_Q(j_i) = 1, \nu_P(j_i) = \nu_Q(j_i) = 0$. In both cases, $G^I_{accy}(P, Q)$ is obviously equal to 1.
(E2). Let $D^I_{acey}(G,H) = 0$.

i.e.,

$$\frac{1}{2h(1-e^{0.5})}\sum_{i=1}^{h}\left[\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)e^{\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)}+\left(\frac{\nu_P(j_i)+1-\mu_P(j_i)}{2}\right)e^{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)}-e^{0.5}\right]$$
$$+\frac{1}{2h(1-e^{0.5})}\sum_{i=1}^{h}\left[\begin{array}{l}\sqrt{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)\times\left(\frac{\mu_Q(j_i)+1-\nu_Q(j_i)}{2}\right)}\times e^{\sqrt{\left(\frac{1+\nu_G(z_i)-\mu_G(z_i)}{2}\right)\times\left(\frac{1+\nu_H(z_i)-\mu_H(z_i)}{2}\right)}}\\+\sqrt{\left(\frac{1+\nu_G(z_i)-\mu_G(z_i)}{2}\right)\times\left(\frac{1+\nu_H(z_i)-\mu_H(z_i)}{2}\right)}e^{\sqrt{\left(\frac{\mu_P(j_i)+1-\nu_P(j_i)}{2}\right)\times\left(\frac{\mu_Q(j_i)+1-\nu_Q(j_i)}{2}\right)}}\\-e^{0.5}\end{array}\right]=0$$

Since there are only positive elements in the previous summation, the previous equation can only be true if $\mu_P(j_i) = \nu_P(j_i), \forall\ j_i \in J$.

Conversely, Let us consider $\mu_P(j_i) = \nu_P(j_i), \forall\ j_i \in J$, which obviously shows $G^I_{accy}(P, Q) = 0$.
(E3). It is simple to show this from Eq. (46).

(E4). It is easy to demonstrate $G^{I}_{accy}(P, Q) = G^{I}_{H}(P)$ for $P = Q$ using the definition from Eq. (46) . Hence $G^{I}_{accy}(P, Q)$ is valid AInF-accuracy measure.

4.1.1. Application of Proposed Accuracy Measure in Pattern Recognition

Now, using an example, we will show how to use the suggested accuracy measure in the pattern recognition issue.

Problem: Consider a finite set $Z = \{z_1, z_2, z_3, \ldots, z_n\}$. Take s patterns, represented by AInF-sets. $X_t = \{<j_i, \mu_{X_t}(j_i), \nu_{X_t}(j_i)>: j_i \in J\}(\mathrm{t} = 1,2,3, \ldots, \mathrm{s})$. Take $C = \{<j_i, \mu_C(j_i), \nu_C(j_i)>: j_i \in J\}$ as anonymous pattern. Our main goal is to fit pattern C into the given pattern X_t. Three methods are employed for identifying patterns:

- **Similarity measure method:** [101] If $F\ (P, Q)$ indicates the similarity between pattern P and pattern Q, then C is identified as pattern *Xt,* where

$$F(C, X_t) = \max_{t=1,2,3,\ldots,s}\big(F(C, X_t)\big).$$

- **Dissimilarity measure method:** [102] If $G(P, Q)$ indicates the dissimilarity between pattern P and pattern Q, then C is identified as pattern *Xt*, where

$$G(C, X_t) = \min_{t=1,2,3,\ldots,s}\big(G(P, X_t)\big).$$

- **Accuracy measure strategy:** If $\mathrm{H}(P, Q)$ indicates the accuracy between pattern P and pattern Q, then C is identified as pattern *Xt*, where

$$H(C, X_t) = \max_{t=1,2,3,\ldots,s}\big(H(C, X_t)\big).$$

While [103] investigated pattern detection using dissimilarity measure, [104] investigated pattern detection using similarity measure. From comparative investigations of similarity and dissimilarity measures, we see that not all pattern identification problems are well-suited for either measure. However, accuracy measure rather than similarity and dissimilarity measures could be a preferable option for handling pattern identification problems. We compare the [104] examples in the pattern recognition problem to demonstrate the effectiveness of the proposed AInF-accuracy measure.

Example 4. Let $J = \{j_1, j_2, j_3\}$ is any finite set. Let D_1, D_2, D_3 are three patterns defined on J as follows:

$$D_1 = \{(j_1, 0.6, 0.1), (j_2, 0.5, 0.2), (j_3, 0.4, 0.3), (j_4, 0.7, 0.2)\};$$

$$D_2 = \{(j_1, 0.5, 0.5), (j_2, 0.5, 0.3), (j_3, 0.6, 0.1), (j_4, 0.8, 0.1)\};$$

$$D_3 = \{(j_1, 0.0, 0.0), j_2, 0.4, 0.2), (j_3, 0.3, 0.3), (j_4, 0.5, 0.4)\}.$$

Consider an unknown pattern C is defined as follows:

$$C = \{(j_1, 0.1, 0.0), (j_2, 0.5, 0.2), (j_3, 0.4, 0.3), (j_4, 0.7, 0.2)\}.$$

The goal is to identify the unknown pattern C among a series of patterns D_1, D_2, D_3. [104]created a similarity measure approach to address this problem using pattern recognition. Table 5 displays the calculated outcomes.

Table 5. Similarity measurement between known and unknown patterns

Similarity measures	$F(C,D_1)$	$F(C,D_2)$	$F(C,D_3)$	Detected/Not detected.
F_C[105]	0.825	**0.788**	**0.788**	Not detected
F_H [106]	0.825	0.863	0.788	Detected as D_1
F_O [107]	0.866	0.846	0.810	Detected as D_1
F_{HB} [108]	0.825	**0.788**	**0.788**	Not detected
F^1_{HY}[89]	**0.975**	**0.975**	0.950	Not detected
F^2_{HY}[89]	**0.961**	**0.961**	0.923	Not detected
F^3_{HY}[89]	**0.951**	**0.951**	0.905	Not detected

F_e^p[109]	0.992	0.981	0.997	Detected as D_3

From Table 5, we discovered the similarity measure F_C [105], F_{HB} [108], F_{HY}^1 [89], F_{HY}^2 [89], F_{HY}^3 [89] are unable to recognise the pattern C, although similarity measure F_H [106], F_O [107] and F_e^p [109] quickly identify the pattern C.

[103] created a similarity measure approach to address this problem using pattern recognition. Table 6 displays the calculated outcomes.

Table 6. Dissimilarity measurement between known and unknown patterns

Dissimilarity measures	$G(C,D_1)$	$G(C,D_2)$	$G(C,D_3)$	Detected/Not detected.
G_{eh} [110]	**0.225**	**0.225**	0.350	Not detected
G_h[111]	**0.225**	**0.225**	0.350	Not detected
G_E [55]	0.235	0.278	0.515	Detected as D_1
G_Z^1 [112]	0.163	0.235	0.325	Detected as D_1
G_Z^2 [112]	**NaN**	**NaN**	**NaN**	Not detected
G_1 [55]	0.194	0.210	0.281	Detected as D_1

From Table 6, we discovered the similarity measure G_{eh}[110], G_h [111] and G_Z^2 [112] are unable to recognise the pattern R, although similarity measure G_E [55], G_Z^1 [112] and G_1 [55] quickly identify the pattern C.

The accuracy measure technique is now employed, and the provided patterns are subjected to the suggested accuracy measure. The results that were calculated are: G_{accy}^1 (C, D_1) = 0.2824, G_{accy}^1 (C, D_2) = 0.2316 and G_{accy}^1 (C, D_3) = 0.1832. Pattern C is classified using the proposed accuracy measure as a component of pattern D1. As a result, for this pattern recognition task, the proposed accuracy measure technique performs well.

5. THE VIKOR TECHNIQUE IN THE CONTEXT OF AINF IS BASED ON PROPOSED KNOWLEDGE AND SIMILARITY MEASURES

We look for an option in a decision-making (DMI) problem that satisfies every requirement. Numerous issues from our daily lives are involved in this. Examples of these challenges include MCDM, MADM, and MCGDM, which involve a set of criteria/attributes, a set of alternatives, a set of experts or trained individuals, a set of weights for the criteria/attributes, and a variable that might modify the preference order.

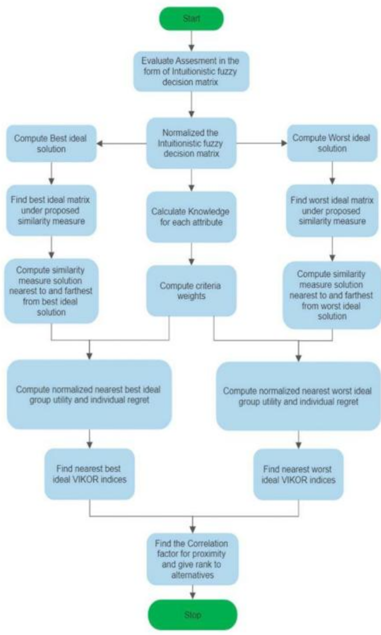

Figure 2. Flowchart showing the phases in the proposed approach

5.1. The Proposed Methodology

[73] examined a method called the VIKOR technique to deal with MCDM difficulties. The “Relative closeness to the best optimal solution” works as a basic building block for this approach. The alternative which is closer to the best optimal solution is the most preferable. After [73] many researchers studied the VIKOR approach and gave its extensions in their ways. In the VIKOR approach, we try to find out a compromise solution to a DMI issue. In the TOPSIS approach the most appropriate alternative is one that is farthest away from the worst optimal solution and closest to the best optimal solution [113]. Both these approaches have their benefits and losses.

5.2. Proposed AInF-Accuracy Based Updated VIKOR Process

Inspired by both the original VIKOR strategy and its adaptations, the modified VIKOR technique based on accuracy for the MCDM issue may be provided by means of the AInF-knowledge measure. Let us consider an MCDM issue. $\Phi = \{\Phi_\alpha\}_{\alpha=1}^{i}$ and $\Psi = \{\Psi_\beta\}_{\beta=1}^{j}$ represents a collection of alternatives and criteria respectively. $\Omega = \{\Omega_\gamma\}_{\gamma=1}^{k}$ is a set of trained persons or experts. Let $\chi = \{\chi_\beta\}_{\beta=1}^{j}$ represents the set of criteria weights with a condition $\sum_{\beta=1}^{j} \chi_\beta = 1$. Figure 2 shows a comprehensive flow chart with all the processes in the recommended strategy. The steps in the suggested methodology are explained as follows:

Step A. Collect all the information and create decision matrix in AInF context: In an intuitionistic fuzzy system, we may create the following decision matrix (Table 7) after receiving the resource person’s responses for a criterion of a certain alternative: where $\mu_{\alpha\beta}$ is the extent to which the Φ_α alternative meets Ψ_β criteria and $v_{\alpha\beta}$ is the extent at which the Φ_α alternative doesn’t satisfy the Ψ_β criteria.

Table 7. Intuitionistic Fuzzy Decision Matrix $k_{i\times j}$

$\kappa_{i\times j}$	Ψ_1	Ψ_2	Ψ_3	...	Ψ_j
Φ_1	$<\mu_{11}, v_{11}>$	$<\mu_{12}, v_{12}>$	$<\mu_{13}, v_{13}>$	...	$<\mu_{1j}, v_{1j}>$
Φ_2	$<\mu_{21}, v_{21}>$	$<\mu_{22}, v_{22}>$	$<\mu_{23}, v_{23}>$	...	$<\mu_{2j}, v_{2j}>$
Φ_3	$<\mu_{31}, v_{31}>$	$<\mu_{32}, v_{32}>$	$<\mu_{33}, v_{33}$	...	$<\mu_{3j}, v_{3j}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Φ_i	$<\mu_{i1}, v_{i1}>$	$<\mu_{i2}, v_{i2}>$	$<\mu_{i3}, v_{i3}>$	...	$<\mu_{ij}, v_{ij}>$

Step B. Compute Normalized decision matrix: Before preceding above, the normalization of the decision matrix k is carried out. Normalized decision matrix τ in the AInF context is obtained as follows:

$$\begin{aligned} \tau &= \{\tau_{\alpha\beta}\}, \\ &= \begin{cases} <\mu_{\alpha\beta}, v_{\alpha\beta}> & Benifit\ criteria \\ <v_{\alpha\beta}, \mu_{\alpha\beta}> & Cost\ criteria \end{cases} \end{aligned} \tag{47}$$

To evaluate the amount of knowledge that was communicated, Eq. (14) is also utilised. Based on benefit and cost criteria, we change out positive and negative participation numbers in this equation. This is due to the fact that the best values for the benefit criteria and the lowest values for the cost criteria are always favoured.

Step C. Calculate criteria weights: The role of Criteria weights in any DM issue is important. The results of any DMI issue depend upon the values of criteria weights. There are two popular approaches to compute criteria weights. They are as follows:

(i.) For unidentified criterion weights: [114] proposed an approach to find out the criteria weights, which is given below:

$$\chi_\beta^N = \frac{1 - AN_\beta}{j - \sum_{\beta=1}^{j} AN_\beta}, \forall\beta = 1,2,3\ldots j; \tag{48}$$

where $AN_\beta = \sum_{\alpha=1}^{i} N(\Phi_\alpha, \Psi_\beta)$ $(\forall\beta = 1,2,3\ldots j)$. $N(\Phi_\alpha, \Psi_\beta)$ is the information given by alternative Φ_α corre-sponding to the criteria Ψ_β. he concepts of information and knowledge measures are complementary to each other. So, in the case of knowledge measures, the criteria weights are computed by the formula given below:

$$\chi_\beta^M = \frac{AM_{\alpha\beta}}{\sum_{\alpha=1}^{i} AM_{\alpha\beta}}, \beta = 1,2,3\ldots j; \tag{49}$$

where $AM_\beta = \sum_{\alpha=1}^{i} M(\Phi_\alpha, \Psi_\beta)$ and $M(\Phi_\alpha, \Psi_\beta)$ is the knowledge passed by alternative Φ_α corresponding to the criteria Ψ_β.

(ii.) For criterion weights that are only partially known: Many real-world issues exist for which none of the qualified individuals are qualified to provide their opinions. There are several causes for it. Time constraints, a lack of understanding of the issue at hand, the application of inappropriate weights to individuals with training, and a narrower perspective on all things are some of the factors that make it impossible to calculate criterion weights properly. All trained individuals therefore select an interval to set criterion weights in order to go over this phase. We use $\bar{E}$ to indicate this interval. The total quantity of knowledge may be calculated using the formula shown below:

$$AM_\beta = \sum_{\alpha=1}^{i} W(\tau_{\alpha\beta}); \tag{50}$$

where

$$\begin{aligned} M(\tau_{\alpha\beta}) &= G_H^l(\Phi_\alpha, \Psi_\beta), \\ &= \frac{1}{h(1 - e^{0.5})}\left[\left(\frac{\mu_{\alpha\beta} + 1 - v_{\alpha\beta}}{2}\right) e^{\left(\frac{v_{\alpha\beta}+1-\mu_{\alpha\beta}}{2}\right)} + \left(\frac{v_{\alpha\beta} + 1 - \mu_{\alpha\beta}}{2}\right) e^{\left(\frac{\mu_{\alpha\beta}+1-v_{\alpha\beta}}{2}\right)} - e^{0.5}\right], \\ &\forall\alpha = 1,2,3\ldots i, \beta = 1,2,3\ldots j. \end{aligned} \tag{51}$$

The formula shown below can be used to obtain criterion weights.

$$\begin{aligned} \max(A) &= \sum_{\beta=1}^{j} (\chi_\beta^M)(AM_\beta), \\ &= \sum_{\beta=1}^{j}\left(\chi_\beta^M \sum_{\alpha=1}^{i} M(\tau_{\alpha\beta})\right), \\ &= \frac{1}{j(1 - e^{0.5})}\sum_{\alpha=1}^{i}\sum_{\beta=1}^{j}\left[\chi_\beta^M \left(\left(\frac{\mu_{\alpha\beta} + 1 - v_{\alpha\beta}}{2}\right) e^{\left(\frac{v_{\alpha\beta}+1-\mu_{\alpha\beta}}{2}\right)} + \left(\frac{v_{\alpha\beta} + 1 - \mu_{\alpha\beta}}{2}\right) e^{\left(\frac{\mu_{\alpha\beta}+1-v_{\alpha\beta}}{2}\right)} - e^{0.5}\right)\right] \end{aligned} \tag{52}$$

where $\chi_\beta^M \in \bar{E}$ and $\sum_{\beta=1}^{j} = 1$.

As a result, the criterion weights determined by Eq. (52) are as follows.

$$argmax(A) = (\chi_1, \chi_2, \ldots, \chi_s)^T; \tag{53}$$

where T is the matrix's transpose.

Step D. Calculate Best/Worst optimal solutions: Now, we determine optimal solution's values. Let $\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_i\}$ represents best optimal solutions and $\theta = \{\theta_1, \theta_2, \ldots, \theta_j\}$ represents worst optimal solutions. We can find their values as follows:

$$\Theta_\beta = \begin{cases} < \max_{\{\alpha\}} \mu_{\alpha\beta}, \min_{\{\alpha\}} v_{\alpha\beta} > \text{Benefit criteria} \\ < \min_{\{\alpha\}} \mu_{\alpha\beta}, \max_{\{\alpha\}} v_{\alpha\beta} > \text{Cost criteria} \end{cases} \tag{54}$$

$$\theta_\beta = \begin{cases} < \min_{\{\alpha\}} \mu_{\alpha\beta}, \max_{\{\alpha\}} v_{\alpha\beta} > \quad Benifit\ criteria \\ < \max_{\{\alpha\}} \mu_{\alpha\beta}, \min_{\{\alpha\}} v_{\alpha\beta} > \quad Cost\ criteria \end{cases} \tag{55}$$

We discover a solution that is closer to the best optimum solution and a solution that is further away from the worst optimal solution for the given problem in these equations.

Step E. Calculate Best/Worst optimal accuracy matrices: Now, using Eq. (46), we assess the accuracy of the best, as well as worst optimal solution w.r.t. each, provided attribute. We can build the best optimal matrix ι and worst optimal matrix ς from these value as follows

$$\iota = \{a_{\alpha\beta}\}_{i\times j}\ and\ \varsigma = \{b_{\alpha\beta}\}_{i\times j} \tag{56}$$

Where $a_{\alpha\beta} = G^I_{accy}(\Theta_\beta, \tau_{\alpha\beta}), b_{\alpha\beta} = G^I_{accy}(\theta_\alpha, \tau_{\alpha\beta})$. In this equation, we find the accuracy of each alternative w.r.t. the best and worst optimal solutions.

Step F. Calculate accuracy vectors: Now, the values of the similarity vectors φ^+ (located extremely close to best optimal solution Θ), φ^- (located extremely far from best optimal solution Θ), υ^+ (located extremely close to worst optimal solution θ), υ^- (located extremely far from worst optimal solution θ) are calculated and their values are given by

$$\begin{aligned} \varpi^+ &= [\varpi^+\varpi^+ \ldots \varpi^+], \varpi^- = [\varpi^-\varpi^- \ldots \varpi^-_j]; \\ v^{\cdot} &= \lfloor v_1 v_2 \ldots v_j \rfloor, v = \lfloor v_1 v_2 \ldots v_j \rfloor. \end{aligned} \tag{57}$$

where $\varpi^+ = \underset{\{\alpha\}}{max}\ a_{\alpha\beta}, \varpi^- = \underset{\{\alpha\}}{min}\ a_{\alpha\beta},\ v^+ = \underset{\{\alpha\}}{max}\ b_{\alpha\beta},\ v^- = \underset{\{\alpha\}}{min}\ b_{\alpha\beta}, (\beta = 1,2,3,\ldots,j)$. These equations find vectors that are extremely close to the best optimal solution and far from the worst optimal solution of the accuracy matrices.

Step G. Create finest and worst normalised collective utility and individual remorse values: The group utility is employed for 'majority', whereas individual regret is employed for 'opponent'. The values of normalized group utility DK_α closest to the best optimal solution and normalized individual regret DG_α closest to the best optimal solution are computed by the formulas given below

$$\begin{aligned} DK_\alpha &= \sum_{\beta=1}^{j} \chi^M_\beta \frac{\varpi^+_\beta - a^+_{\alpha\beta}}{\varpi^+_\beta - \varpi^-_\beta}, \\ DG_\alpha &= \max_{\{\beta\}} \left(\chi^M_\beta \frac{\varpi^+_\beta - a^+_{\alpha\beta}}{\varpi^+_\beta - \varpi^-_\beta} \right). \forall \alpha = 1,2,3\ldots, i. \end{aligned} \tag{58}$$

Similarly, the values of normalized group utility DK_α closest to the worst optimal solution and normalized individual regret DG_α closest to the worst optimal solution are computed by the formulas given below

$$\begin{aligned} GK_\alpha &= \sum_{\beta=1}^{j} \chi^M_\beta \frac{v^+_\beta - b^+_{\alpha\beta}}{v^+_\beta - v^-_\beta}, \\ GR_\alpha &= \max_{\{\beta\}} \left(\chi^M_\beta \frac{v^+_\beta - b^+_{\alpha\beta}}{v^+_\beta - v^-_\beta} \right). \end{aligned} \tag{59}$$

Step H. Compute closest Best/Worst optimal VIKOR indices: VIKOR indices tell us about the relative closeness to the optimal solution. We measure the relative closeness/farness w.r.t Best/Worst optimal solutions. Now, we calculate the values of best optimal VIKOR indices Z^+ (located extremely close to the best optimal solution) and worst optimal VIKOR indices Z^- (located extremely far from the best optimal solution) by using the formulas

$$Z_\alpha^+ = \delta \frac{DK_\alpha - DK^*}{DK^- - DK^*} + (1-\epsilon)\frac{DG_\alpha - DG^*}{DG^- - DG^*},$$
$$Z_\alpha^- = \delta \frac{GK_\alpha - GK^*}{GK^- - GK^*} + (1-\epsilon)\frac{GR_\alpha - GR^*}{GR^- - GR^*}; \tag{60}$$

$$where\ DK^- = \max_\alpha DK_\alpha, DK^* = \min_\alpha DK_\alpha, DG^- = \max_\alpha DG_\alpha, DG^* = \min_\alpha DG_\alpha;$$
$$GK^- = \max_\alpha TK_\alpha,\ GK^* = \min_\alpha GK_\alpha, GR^- = \max_\alpha GR_\alpha, GR^* = \min_\alpha GR_\alpha.$$

The figures of ϵ and $(1-\epsilon)$, respectively, indicate the relative importance of the strategies of ”the vast majority of attribute” as well as ”the individual remorse”. In most cases, the worth of $\epsilon = 0.5$ being used.

Step I. Compute proximity factor: In the end, the proximity factors related to each alternative, are calculated by using the formula given below:

$$GP_\alpha = \frac{Z_\alpha^+}{Z_\alpha^+ + Z_\alpha^-}, \forall \alpha = 1,2,3\dots, i. \tag{61}$$

We provide the proximity factors in ascending order after calculating their values for each alternative. The proximity factor with a lesser value is selected for an alternative with greater effectiveness.

5.3. Case Study

The urethra serves as a conduit for urine excretion but also provides a potential entry point for microorganisms, par- ticularly infectious agents, into the urinary system. While bacteria colonize the urethra and reside near its entrance in both men and women, they are typically flushed out during micturition. Given the anatomical structure of the human body, it is unsurprising that urinary tract infections (UTIs) are among the most prevalent bacterial infections. However, considering the constant exposure of the urinary system to microbial threats, it is remarkable that UTIs are not even more frequent. The progression of asymptomatic bacterial colonization into symptomatic infection is influ- enced by both host and bacterial factors. Host-related factors include anatomical or functional abnormalities, genetic predisposition, and behaviors that increase exposure to uropathogens or facilitate their migration into the bladder. Given the widespread occurrence of bacteriuria and urinary symptoms in the general population, it is possible for an individual without a UTI to exhibit both conditions simultaneously by coincidence. This overlap can lead to over- diagnosis, particularly when clinical decisions are based solely on the presence of symptoms or the results of a dipstick urinalysis. Some common Symptoms of UTIs are represented by Figure 3. The initial line of treatment for urinary tract infections is usually antibiotics. The bacteria that cause the sickness are eliminated by these medications. It is imperative that you take them precisely as prescribed by your physician. If you don’t, a minor UTI might grow into a potentially fatal blood or kidney infection. To confirm that you have a UTI, our doctor will take a sample of your urine. The type of bacteria you have will then be ascertained by growing the germs in a dish for a few days. We refer to this as a culture. Which medication is administered and for how long depends on the kind of medication used as well as the germs detected in your urine. “Complicate” indicates that you have a urinary tract ailment or condition. You might have a constriction of your ureters, which are the tubes that transport urine from your kidneys to your bladder, a restriction of the urethra, which carries pee from the bladder out of your body, or a blockage such as a kidney stone or an enlarged prostate (in males). It’s also possible that you have a diverticulum or urine fistula inside your bladder. A complicated infection may require a higher dosage of antibiotics. Should the infection have progressed to your kidneys, you could require hospitalisation for treatment with potent antibiotics. Various characteristics of antibiotics exist, contingent on the physician prescribing the most appropriate medication for an individual. The type of bacteria you have will then be ascertained by growing the germs in a dish for a few days. We refer to this as a culture. Which medication is administered and for how long depends on the kind of medication used as well as the germs detected in your urine. The antibiotic should be chosen using the features provided in Figure 4.

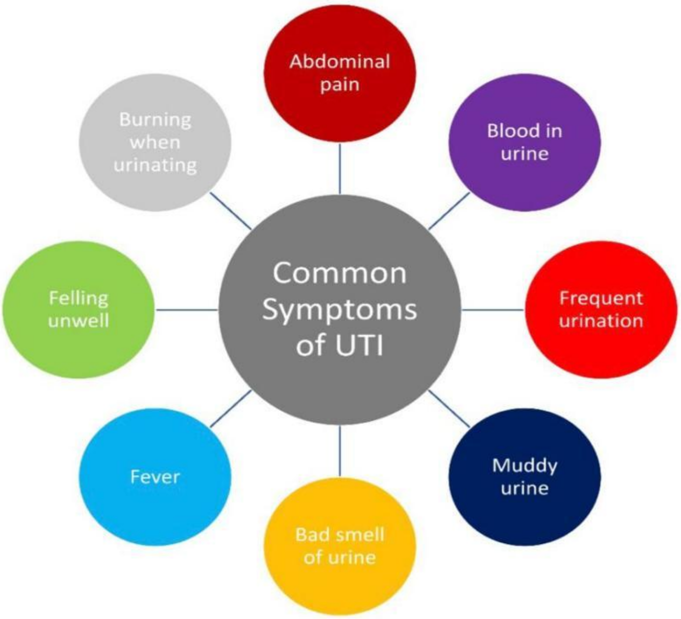

Figure 3. Common Symptoms of UTI.

Figure 4. Features of the best antibiotic.

In this paper, we take five antibiotics recommended by professional doctors to treat UTIs. They are represented by Φ_1, Φ_2, Φ_3, Φ_4, Φ_5 as alternatives. They are-**(i.)** Amoxicillin, **(ii.)** Ceftriaxone, **(iii.)** Levofloxacin, **(iv.)** Ni- trofurantoin, and **(v.)** Cephalexin. To select the best antibiotic, we take ten criteria, which are represented by Ψ_1, Ψ_2, Ψ_3, Ψ_4, Ψ_5, Ψ_6, Ψ_7, Ψ_8, Ψ_9, Ψ_{10} and are given by-**(i.)** Effectiveness, **(ii.)** Side-effects, **(iii.)** Fast-acting, **(iv.)** Cost, **(v.)** Easily available, **(vi.)** Antibacterial resistance, **(vii.)** Prevents further infection, **(viii.)** Allergic Reactions, **(ix.)** Easy to take, and **(x.)** Duration of taking dosages. The main definitions of these criteria are given in Table 8. Here, Effectiveness, Fast-acting, Easily available, Prevents further infection, and Easy to take are beneficial criteria (Bigger value is desirable), whereas Side-effects, Cost, Antibacterial resistance, Allergic Reactions, and Duration of taking dosages are cost criterion (Smaller value is desirable). They are represented by Figure 5. There are ten trained persons involved in this DMI issue, which are represented by $\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6, \Omega_7, \Omega_8, \Omega_9, \Omega_{10}$. The primary struc- ture of the given MCDM issue is given in Figure 6. Now, we make use of the suggested method to opt for the best antibiotic. The proposed approach follows these steps:

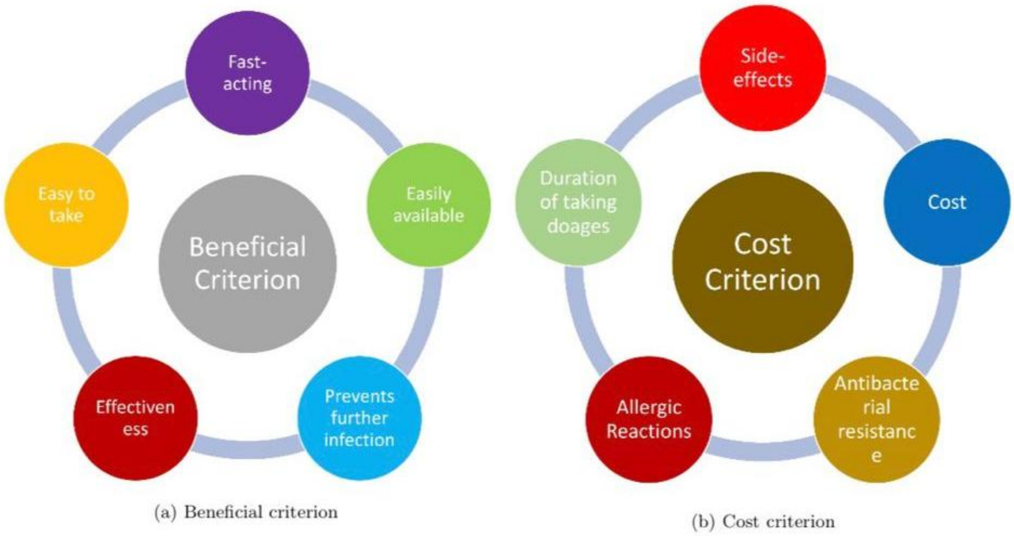

Figure 5. Beneficial and Cost Criterion.

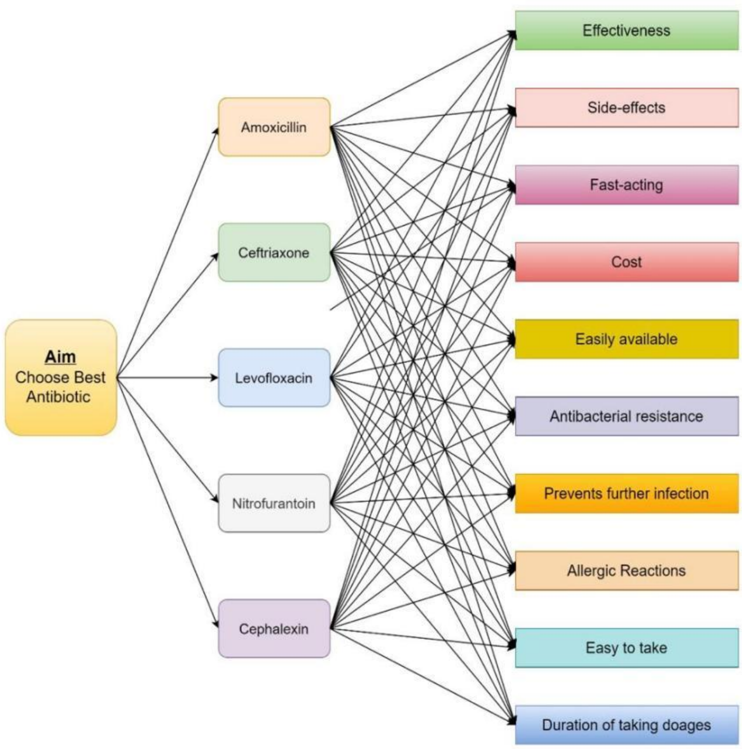

Figure 6. Primary structure of the given MCDM issue.

Table 8. Definition of Criteria

Criteria	Definition
Effectiveness (Ψ1)	Any antibiotic medication's efficacy or effectivity in treating a urinary tract infection is measured by its ability to destroy germs. The quicker the medication clears the UTI, the more successful it is.
Side-effects (Ψ2)	A side-effect is an adverse medication reaction that goes beyond what is anticipated. Diarrhoea is among the adverse effects that can occur from antibiotics. These side effects are usually mild and should go away when your therapy session is over. The antibiotic of choice is the one with fewer adverse effects.
Fast-acting (Ψ3)	The term "fast-acting medication" describes a medication that acts on an infection quickly. The infection will be treated faster if the medication starts working sooner.
Cost (Ψ4)	Antibiotic medicine costs are expressed as the price that is either paid or levied. All antibiotic medications should be more reasonably priced so that everyone may afford them.
Easily available (Ψ5)	Because medications are readily available, they may be purchased from any store with ease. Having more accessible and simpler access to antibiotic medications is always preferable.
Antibacterial resistance (Ψ6)	Antibacterial resistance occurs when microorganisms such as bacteria and fungi gain the capacity to resist medications that are meant to kill them. Any antibiotic medicine to treat UTIs should be selected in such a way that it must have low antibacterial resistance.
Prevents further infection (Ψ7)	A necessary characteristic of every antibiotic is its ability to stop the illness from spreading deeper within the body; without this ability, an antibiotic won't be able to treat the infection.
Allergic Reactions (Ψ8)	When the body's immune system reacts in excess to a harmless chemical called an allergen, an allergic reaction ensues. Most allergic reactions are mild to severe. Some are- wheezing, coughing, itchy skin rash, Tightness in the throat, which can make it difficult to breathe.
Easy to take (Ψ9)	There are many ways to take an antibiotic medicine. They are Lollipops, Lip Balm, Topical Ointment, Orifice Drops, Powder Form, Liquid, etc. The best way is the way with which you are comfortable.
Duration of taking dosages (Ψ10)	Duration of taking dose refers to the time for which an antibiotic is safe to be taken to treat UTI. Any medicine should not be taken for a long time. So, for a minimum time duration, it should be taken.

Case 1: For unidentified criterion weights:

Step A: We gather every resource person's input about a criterion related to a certain choice. Table 9 displays the decision matrix that is produced by combining the responses provided by each resource person. The matrix $\tau = \tau_{\alpha\beta} =< \mu_{\alpha\beta}, \nu_{\alpha\beta} >$, $\mu_{\alpha\beta}$ indicates the percentage of resource people who favour alternative Φ_α in comparison to criterion Ψ_β for all resource people involved. On the other hand, $\nu_{\alpha\beta}$ represents the percentage of all resource people who oppose alternative Φ_α in light of criteria Ψ_β for all resource people involved. The degree of knowledge that each particular criterion was able to evaluate is also shown in Table 9.

Table 9. Intuitionistic Fuzzy Decision Matrix $\kappa_{5\times10}$

$\kappa_{5\times10}$	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}
Φ_1	$<0.4,0.4>$	$<0.6,0.1>$	$<0.3,0.5>$	$<0.6,0.2>$	$<0.4,0.3>$	$<0.6,0.2>$	$<0.4,0.3>$	$<0.3,0.4>$	$<0.1,0.6>$	$<0.5,0.3>$
Φ_2	$<0.6,0.3>$	$<0.5,0.4>$	$<0.6,0.3>$	$<0.5,0.2>$	$<0.8,0.2>$	$<0.6,0.4>$	$<0.5,0.3>$	$<0.7,0.1>$	$<0.5,0.5>$	$<0.3,0.6>$
Φ_3	$<0.6,0.3>$	$<0.5,0.3>$	$<0.4,0.4>$	$<0.3,0.5>$	$<0.5,0.4>$	$<0.6,0.4>$	$<0.6,0.3>$	$<0.4,0.4>$	$<0.7,0.1>$	$<0.5,0.4>$
Φ_4	$<0.4,0.4>$	$<0.7,0.3>$	$<0.5,0.4>$	$<0.6,0.4>$	$<0.3,0.4>$	$<0.7,0.2>$	$<0.3,0.5>$	$<0.4,0.6>$	$<0.5,0.4>$	$<0.4,0.4>$
Φ_5	$<0.5,0.5>$	$<0.4,0.4>$	$<0.5,0.5>$	$<0.5,0.3>$	$<0.3,0.4>$	$<0.3,0.3>$	$<0.5,0.4>$	$<0.4,0.4>$	$<0.3,0.5>$	$<0.3,0.3>$

Step B: In AInF context, the Normalised decision matrix τ may be constructed using Eq. (47) and is presented in Table 10. Also, the knowledge obtained from these criteria is shown in Table 10.

Table 10. Normalized Decision Matrix in AInF-context $\tau_{5\times10}$.

$\tau_{5\times10}$	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ψ_5	Ψ_6	Ψ_7	Ψ_8	Ψ_9	Ψ_{10}

Φ_1	$<0.4,0.4>$	$<0.6,0.1>$	$<0.3,0.5>$	$<0.2,0.6>$	$<0.4,0.3>$	$<0.2,0.6>$	$<0.4,0.3>$	$<0.4,0.3>$	$<0.1,0.6>$	$<0.3,0.5>$
Φ_2	$<0.6,0.3>$	$<0.4,0.5>$	$<0.6,0.3>$	$<0.2,0.5>$	$<0.8,0.2>$	$<0.4,0.6>$	$<0.5,0.3>$	$<0.1,0.7>$	$<0.5,0.5>$	$<0.6,0.3>$
Φ_3	$<0.6,0.3>$	$<0.3,0.5>$	$<0.4,0.4>$	$<0.5,0.3>$	$<0.5,0.4>$	$<0.4,0.6>$	$<0.6,0.3>$	$<0.4,0.4>$	$<0.7,0.1>$	$<0.4,0.5>$
Φ_4	$<0.4,0.4>$	$<0.3,0.7>$	$<0.5,0.4>$	$<0.4,0.6>$	$<0.3,0.4>$	$<0.2,0.7>$	$<0.3,0.5>$	$<0.4,0.6>$	$<0.5,0.4>$	$<0.4,0.4>$
Φ_5	$<0.5,0.5>$	$<0.4,0.4>$	$<0.5,0.5>$	$<0.3,0.5>$	$<0.3,0.4>$	$<0.3,0.3>$	$<0.5,0.4>$	$<0.4,0.4>$	$<0.3,0.5>$	$<0.3,0.3>$
G_H^I	0.0345	0.0886	0.0268	0.0710	0.3137	0.0943	0.1152	0.1581	0.1276	0.0268

Step C: Eq. (49) is used to find Criteria weights. Their values are given below

$$\chi = \{0.0326, 0.0838, 0.0253, 0.0671, 0.2968, 0.0892, 0.1090, 0.1496, 0.1207, 0.0253\}.$$

Step D: The values of best optimal solutions and worst optimal solutions are calculated by using Eqs. (54) and (55) and are given by

$\Theta = \{< 0.6, 0.3 >, < 0.7, 0.1 >, < 0.6, 0.3 >, < 0.6, 0.2 >, < 0.8, 0.2 >, < 0.7, 0.2 >, < 0.6, 0.3 >, < 0.7, 0.1 >, < 0.7, 0.1 >, < 0.5, 0.3 >\}$.
$\theta = \{< 0.4, 0.5 >, < 0.4, 0.4 >, < 0.3, 0.5 >, < 0.3, 0.5 >, < 0.3, 0.5 >, < 0.3, 0.4 >, < 0.3, 0.5 >, < 0.3, 0.6 >, < 0.1, 0.6 >< 0.3, 0.6 >\}$.

Step E: Best optimal matrix ι and Worst optimal matrix ς are calculated by using Eq. (56) as follows

$$\iota = \begin{bmatrix} 0.8950 & 0.8669 & 0.9421 & 1 & 0.9718 & 0.9007 & 0.9069 & 0.6109 & 0.8669 & 0.9718 \\ 1 & 0.6109 & 1 & 0.9204 & 1 & 0.7322 & 0.9421 & 1 & 1 & 0.5990 \\ 1 & 0.6461 & 0.8950 & 0.8626 & 0.9715 & 0.7322 & 1 & 0.5990 & 0.8950 & 1 \\ 0.8950 & 0.7039 & 0.9069 & 0.8315 & 0.9718 & 1 & 0.9421 & 0.6150 & 0.9069 & 0.9421 \\ 0.8950 & 0.5990 & 0.9421 & 0.8626 & 0.9718 & 0.7162 & 0.9069 & 0.599 & 0.7162 & 0.9069 \end{bmatrix} \text{and}$$

$$\varsigma = \begin{bmatrix} 0.9861 & 0.7162 & 1 & 0.8626 & 0.9648 & 0.8274 & 0.9648 & 0.9069 & 0.9069 & 0.9881 \\ 0.9069 & 0.9881 & 0.9421 & 0.9430 & 0.9930 & 0.9959 & 1 & 0.7040 & 0.9881 & 0.9069 \\ 0.9069 & 0.9529 & 0.9524 & 1 & 0.9648 & 0.9959 & 0.9421 & 0.8950 & 0.9648 & 1 \\ 0.9881 & 0.8951 & 0.9648 & 0.9648 & 0.7281 & 1 & 0.9110 & 1 & & 0.9648 \\ 0.9881 & 1 & 1 & 0.8626 & 0.9648 & 0.9881 & 0.9648 & 0.8940 & 0.7633 & 0.9421 \end{bmatrix}$$

Step F: The accuracy vectors are determined by using Eq. (57) and aeprovided by

$$\varpi+ = \{1, 0.8699, 1, 1, 1, 1, 1, 1, 1, 1\},$$
$$\varpi- = \{0.8950, 0.5990, 0.8950, 0.8155, 0.9715, 0.7162, 0.9069, 0.5990, 0.5990\},$$
$$\upsilon+ = \{1, 1, 1, 1, 1, 0.9959, 1, 0.9110, 1, 1\},$$
$$\upsilon- = \{0.9069, 0.7162, 0.9421, 0.8626, 0.9648, 0.7281, 0.9421, 0.7040, 0.9069\}.$$

Step G: The virtues of normalised closest finest ideal collective efficiency DK_α and normalised nearest finest ideal individual remorse DG_α for each option are determined using Eq. (58) and are displayed below

$$DK_1 = 0.5838,\ DK_2 = 0.3695,\ DK_3 = 0.8529,\ DK_4 = 0.7733,\ DK_5 = 0.8874;$$
$$DG_1 = 0.2613,\ DG_2 = 0.1676,\ DG_3 = 0.2641,\ DG_4 = 0.2611,\ DG_5 = 0.2613.$$

For each alternative, the estimated values of normalised closest worst ideal group utility $GK\alpha$ and normalised near worst ideally individuals regret $GR\alpha$ are presented below using Eq. (59)

$$GK_1 = 0.6197,\ GK_2 = 0.4872,\ GK_3 = 0.5534,\ GK_4 = 0.5799,\ GK_5 = 0.3433;$$
$$GR_1 = 0.2641,\ GR_2 = 0.1992,\ GR_3 = 0.2641,\ GR_4 = 0.2641,\ GR_5 = 0.2641.$$

Step H: The outcomes of the VIKOR indexes Z+ and Z− for each option, as determined by Eq. (60), are displayed below

$$Z_1^+ = 0.6922, Z_2^+ = 0, Z_3^+ = 0.9666, Z_4^+ = 0.5946, Z_5^+ = 0.9854;$$
$$Z_1^- = 1, Z_2^- = 0.4639, Z_3^- = 0.8908, Z_4^- = 0.9344, Z_5^- = 0.5449.$$

Step I: Using Eq. (61) the estimated correlation coefficient GP_α readings for each possibility are displayed below

$$GP_1 = 0.4090,\ GP_2 = 0,\ GP_3 = 0.5446,\ GP_4 = 0.3888,\ GP_5 = 0.6439.$$

By applying the suggested accuracy measure in Table 11, we construct the values of the nearest finest ideal VIKOR indices Z_a^+, closest worse ideal VIKOR indices Z_a^-, correlation factor GP_α, and rankings for each alternative and all the alternatives with attained ranks are shown with the help of Figure 7. According to, these alternatives are ranked in order of preference $\Phi_2 > \Phi_4 > \Phi_1 > \Phi_3 > \Phi_5$. We now do a sensitivity analysis with regard to various values of weightage (ϵ) .The range ϵ value is 0 to 1. We take the various values of ϵ, ranging from 0 to 1, with a 0.1 step interval. For various choices of ϵ, the correlation factor values in accordance with the suggested accuracy measure are displayed in Table 12.

Table 11. Generated Ranks, Correlation coefficient, and VIKOR indexes.

Alternative↓	←Accuracy measure →			Ranking
	Z_a^+	Z_a^-	GP_α	
Φ_1	0.6922	1	0.4090	3
Φ_2	0	0.4639	0	1
Φ_3	0.9666	0.8908	0.5446	4
Φ_4	0.5946	0.9344	0.3888	2
Φ_5	0.9854	0.5449	0.6439	5

Table 12. Sensitive study for various ϵ values using the accuracy measure.

Weightage (ϵ)↓	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	*Preference Order*	*finest alternative*
$\epsilon=0$	0.4926	0	0.5	0.4921	0.4926	$\phi_2 > \phi_4 > \phi_1 = \phi_5 > \phi_3$	ϕ_2
$\epsilon=0.1$	0.47783	0	0.5037	0.4904	0.5172	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.2$	0.4621	0	0.5076	0.4886	0.5442	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.3$	0.4454	0	0.5037	0.4868	0.5741	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.4$	0.4277	0	0.5157	0.4849	0.6071	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.5$	0.4090	0	0.5446	0.3888	0.6439	$\phi_2 > \phi_4 > \phi_1 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.6$	0.3887	0	0.5244	0.4809	0.6853	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.7$	0.3670	0	0.5289	0.4788	0.7320	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.8$	0.3438	0	0.5336	0.4766	0.7852	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=0.9$	0.3188	0	0.5384	0.4744	0.8464	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2
$\epsilon=1$	0.2919	0	0.5434	0.4721	0.5175	$\phi_2 > \phi_1 > \phi_4 > \phi_3 > \phi_5$	ϕ_2

Case 2: For criterion weights that are only partially known:

Because of some real-world problems, trained persons are not able to give a whole number to the criteria weights. So, we use intervals instead of an exact number for criteria weights. Thus, take a DMI issue of partially known criteria weights. The intervals for criteria weights provided by the trained persons are given below:

$$\bar{E} = \begin{cases} 0.09 \leq \chi_1^M \leq 0.11, 0.08 \leq \chi_2^M \leq 0.11, 0.09 \leq \chi_3^M \leq 0.11, 0.10 \leq \chi_4^M \leq 0.13, 0.07 \leq \chi_5^M \leq 0.09 \\ 0.06 \leq \chi_6^M \leq 0.09, 0.10 \leq \chi_7^M \leq 0.12, 0.10 \leq \chi_8^M \leq 0.12, 0.10 \leq \chi_9^M \leq 0.12 0.06 \leq \chi_{10}^M \leq 0.08. \end{cases} \tag{62}$$

$A_{\max} = 0.0326\chi_1^M + 0.0838\chi_2^M + 0.0253\chi_3^M + 0671\chi_4^M + 0.2968\chi_5^M + 0.0892\chi_6^M + 0.1090\chi_7^M + 0.1496\chi_8^M + 0.1207\chi_9^M + 0.0253\chi_{10}^M$;

with conditions

$$\begin{cases} 0.09 \leq \chi_1^{\mathrm{M}} \leq 0.11, \\ 0.08 \leq c_2^M \leq 0.11 \\ 0.09 \leq c_3^M \leq 0.11, \\ 0.010 \leq c_4^M \leq 0.13, \\ 0.07 \leq c_5^M \leq 0.09, \\ 0.06 \leq c_6^M \leq 0.09, \\ 0.10 \leq c_7^M \leq 0.12, \\ 0.10 \leq c_8^M \leq 0.12, \\ 0.10 \leq c_9^M \leq 0.12, \\ 0.06 \leq c_8^M \leq 0.08. \\ \sum_{\alpha=1}^{10} \chi_\alpha^M = 1. \end{cases} \tag{63}$$

Eq. (63), which is solved using MATLAB software, yields the following result:

$$\chi_1^M = 0.11, \chi_2^M = 0.116, \chi_3^M = 0.11, \chi_4^M = 0.11, \chi_5^M = 0.09, \chi_6^M = 0.09, \chi_7^M = 0.10, \chi_8^M = 0.10, \chi_9^M = 0.10, \chi_{10}^M = 0.08. \tag{64}$$

Again, by solving in the same way as in instance (1), we are able to determine Φ2 as the most desirable option.

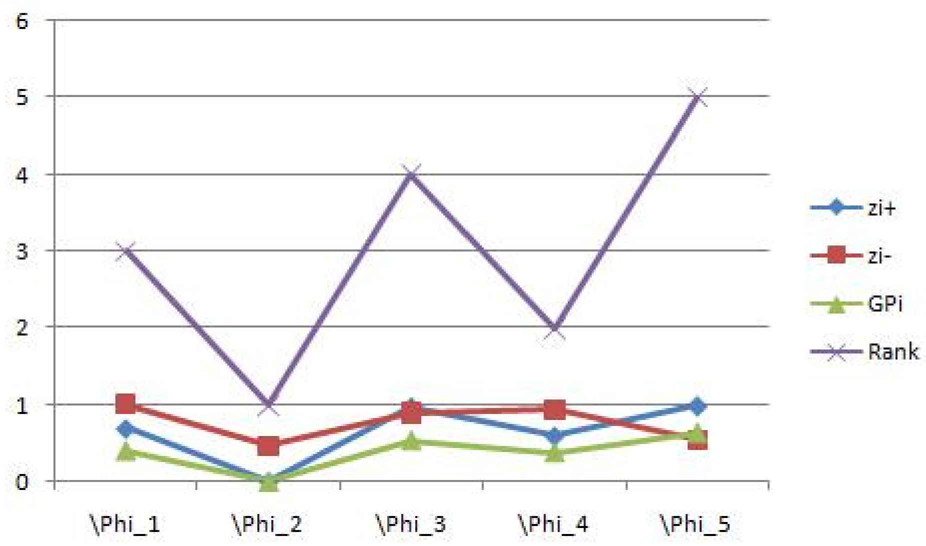

Figure 7. Nearest finest ideal VIKOR indices, closest worse ideal VIKOR indices, correlation factor, and rankings

5.4. Comparison and Analysis

Now, we contrast the proposed decision-making method(DMMs) with other approaches. There are many approaches used for solving MCDM problems for AInF-sets. We take the above case study and solve it with these approaches. There are the following approaches with which we contrast the proposed approach for reliability:

i. [71] proposed TOPSIS (Technique for Order Preference by Similarity to Ideal Solutions) strategy.

ii. [115] suggested Decision-making method(DMMs).

iii. [100] suggested DMMs.

iv. [116] suggested DMMs by utilising several knowledge measures.

v. [117] suggested DMMs by utilising several knowledge measures.

vi. [117] suggested DMMs by the use of knowledge measures looked at by [67].

vii. [117] suggested DMMs by the use of knowledge measures looked at by [59].

A brief comparison with these given approaches is given in Table 13 and Figure 8. The best choice is the one that is closest to the best response and furthest from the worst answer, according to the TOPSIS approach. [118] noted that when comparing the TOPSIS technique with the VIKOR approach, it isn't always the case that the choice most similar to the best solution is also the one most dissimilar to the worst solution. [115] only took into account the relationships between the ideal substitute and alternative natives. Being near the best response could be useful in some situations, but not always because it could make you forget important details. As a result, the output that [115] method suggests is not very trustworthy. To address MCDM challenges in an uncertain environment, [100] developed an approach based on the weighted intuitionistic fuzzy inaccuracy measure. [116] recommended adopting three alternate KM to overcome MCDM issues. By utilising four distinct strategies, [117] offered a way to solve the MCDM problem. In order to solve the related MCDM problem, he also uses the techniques published by [67] and [59]. The problem that is being given has five possible solutions; Table 13 indicates that the Φ2 choice is the best answer based on all of the techniques that have been offered. Consequently, the results of the suggested method may be relied upon.

Table 13. Comparative Table.

Approaches	Preference order	finest alternative
TOPSIS [71]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
DMS [115]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
DMS [100]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^1$ [116]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^2$ [116]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^3$ [116]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^1$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^2$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^3$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs^4$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
$DMMs$ [117]	$\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3 > \Phi_5$	Φ_2
The Proposed method	$\Phi_2 > \Phi_4 > \Phi_1 > \Phi_3 > \Phi_5$	Φ_2

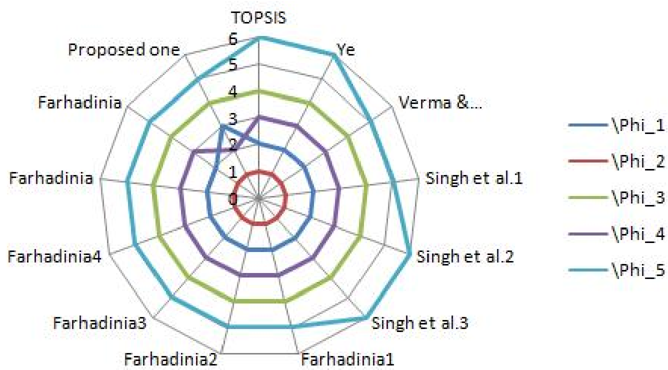

Figure 8. Comparison of the suggested method with other well-known method

6. CONCLUSION

Prior to analysing the characteristics of the suggested measures, the paper presents an exponentially based knowledge measure of AInF-sets. The utility of the proposed measure is illustrated by means of numerical examples where it is juxtaposed with current related measures of AInF-sets. The proposed measure is used to produce an accuracy mea- sure, which is then verified.cIn pattern detection problems, the suggested accuracy measure is applied. A numerical illustration of a pattern recognition problem is examined to verify the efficacy of the suggested accuracy measures. Additionally, a VIKOR method for resolving MCDM problems is offered. This method is based on the suggested Knowledge and Accuracy measure. The key advantage of using the suggested approach is that it allows us to choose the best alternative that fits all criteria. Furthermore, the proposed approach explains why certain alternatives are su- perior to others in solving DMI issues. The proposed approach eliminates the need for time-consuming computations. Furthermore, the suggested approach is used to determine the appropriate antibiotic medicine for treating UTIs. A sensitivity analysis for various weightage values is also provided. To show the efficacy of the suggested approach, a comparison with many other popular approaches is provided. Hesitant Fuzzy set; Interval-valued Intuitionistic Fuzzy set; Picture Fuzzy set; and Neutrosophic Fuzzy set are all covered by the proposed measure's extension. The knowledge and accuracy that are indicated can be used in a variety of contexts, such as feature identification, speech recognition, and picture thresholding.

Limitations: While the suggested exponentially based knowledge and accuracy measures have great benefits, there are some drawbacks. First, the technique is based on established parameter settings, which may not be relevant across all domains. The selection of weightage values in sensitivity analysis may involve subjectivity, necessitating further confirmation by expert agreement. Furthermore, the computational efficiency of the suggested technique may vary while working with large-scale datasets, demanding further improvement. Furthermore, although the approach reduces time-consuming calculations in certain applications, its performance should be compared to real-time decision-making situations. Finally, although the work focuses mostly on theoretical validation and numerical examples, actual case studies in real-world applications would enhance the conclusions.

Future Scope: Future research should focus on refining the parameter selection process to reduce subjectivity and improve generalizability across different domains. The proposed method can be further enhanced by integrating ma- chine learning techniques to optimize weightage values dynamically. Additionally, exploring real-world applications and conducting empirical studies will provide more concrete evidence of its effectiveness. Another promising direction is extending the approach to handle multi-objective decision-making scenarios with greater computational efficiency. Furthermore, the development of hybrid methodologies that incorporate other decision-making frameworks may en- hance the robustness of the proposed measure. Finally, interdisciplinary collaborations with domain experts can ensure practical applicability in diverse fields such as healthcare, finance, and engineering.

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