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ARTICLE

Large Language Model Driven Distributed Robust Multi-Criteria Optimization Decision Method with Interval Binary Linguistic Considering Uncertain Cognitive Preferences — A Case of Heterogeneous Stakeholder Collaborative System for Trans-Regional Carbon Governance

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Abstract

Trans-regional carbon governance is a complex system with diverse stakeholders, conflicting indicators, fuzzy cognition and multi-source uncertainties. Traditional MCDM methods use manual weighting and single-agent frameworks, which cannot accurately capture stakeholders' vague preferences or withstand external disturbances. To address these limitations, this paper proposes a distributed robust multi-criteria optimization decision method combining interval binary linguistic information and large language model (LLM). First, interval binary linguistic variables describe stakeholders' dual uncertain cognition in carbon governance. We design fusion operators to reduce information loss during evaluation aggregation.

Secondly, an implicit preference mining module based on LLM is designed to quantitatively extract risk preference, fairness preference and bounded rationality of decision-makers, so as to realize dynamic and intelligent weight optimization instead of subjective manual weighting. Thirdly, a distributed robust multi-objective optimization model is established for heterogeneous regional governance subjects to balance economic development, carbon emission reduction and governance cost under multiple uncertainties. Finally, a practical case of trans-regional carbon collaborative governance is used for empirical verification, and comparative analysis with classic multi-criteria decision methods is conducted. Results verify its high robustness and practicability, offering a novel framework for intelligent optimization of complex public governance systems.

Keywords: Interval binary linguistic; Large language model; Distributed robust optimization; Multi-criteria decision-making; Trans-regional carbon governance; Heterogeneous stakeholders

1. INTRODUCTION

With the continuous advancement of global low-carbon development strategies, trans-regional carbon governance has become a core task for promoting carbon peaking and carbon neutrality goals. Carbon governance coordinates multiple regions, authorities and stakeholders, leading to complex practical challenges.

On the one hand, each regional subject has different development demands and resource endowments, leading to obvious conflicts among economic benefit, carbon emission reduction target and public livelihood in the decision-making process. On the other hand, the decision-makers' understanding of governance indicators is often ambiguous and uncertain due to the influence of policy dynamics, market fluctuations and bounded rationality. In addition, the traditional centralized decision mode is difficult to adapt to the distributed operation characteristics of multi-region and multi-stakeholder, which easily causes unreasonable resource allocation and low governance efficiency [1].

Multi-criteria decision-making (MCDM) is one of the mainstream tools to solve complex evaluation and optimization problems, and has been widely applied in energy, environment and public management fields [2]. A large number of existing studies adopt intuitionistic fuzzy sets, hesitant fuzzy sets and other fuzzy information models to describe uncertain decision information, and improve classic methods such

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as TOPSIS and VIKOR to adapt to different decision scenarios. However, most of the current fuzzy decision models focus on single-dimensional uncertain information description, and cannot effectively reflect the dual expression characteristics of decision-makers' positive and negative cognition in complex public decision-making. Meanwhile, the criterion weights in most studies are determined by expert scoring or fixed weighting rules, which is highly subjective and cannot dynamically adjust with the change of decision-makers' implicit preferences [3].

In recent years, artificial intelligence represented by large language models has achieved remarkable results in information extraction, semantic understanding and preference analysis, bringing new ideas for the innovation of decision science and optimization technology. Many scholars have begun to combine machine learning and deep learning algorithms with optimization models to construct data-driven intelligent decision frameworks. Existing AI applications mostly target scheduling, supply chain and index prediction. Few studies extract implicit preferences via LLMs and embed them into distributed robust optimization, lacking complete theoretical and practical frameworks.

Robust optimization is an effective method to deal with system uncertainty, which can ensure that the decision scheme still maintains good performance under the disturbance of various uncertain factors. At present, robust optimization has been gradually introduced into carbon emission reduction and energy system scheduling research, but most studies adopt single-agent robust models, ignoring the game and collaborative relationship between heterogeneous subjects in trans-regional governance [4]. How to combine distributed decision mechanism with robust optimization, and further couple with fuzzy information expression and intelligent preference mining technology, has become an important research gap in the field of complex system decision optimization.

In view of the above research deficiencies, this paper takes trans-regional carbon governance heterogeneous collaborative system as the research object, and constructs a novel distributed robust multi-criteria optimization decision method driven by large language model and interval binary linguistic information. The main research work of this paper is arranged as follows: Section 2 reviews the relevant literature and systematically sorts out the research progress and existing deficiencies of fuzzy multi-criteria decision-making, AI-driven decision optimization and carbon governance decision. Section 3 introduces the basic definitions and related theories of interval binary linguistic variables, distributed decision and robust optimization, which lays a theoretical foundation for subsequent model construction. Section 4 elaborates the proposed method in detail, including interval binary linguistic information fusion operator, LLM-based implicit preference mining mechanism, and distributed robust multi-criteria optimization model as well as its solution algorithm. Section 5 carries out empirical analysis based on actual trans-regional carbon governance cases, and verifies the effectiveness and superiority of the proposed method through comparative experiments and sensitivity analysis. Section 6 discusses the application value, applicable scenarios and limitations of the method. Section 7 summarizes the full text and puts forward the direction of future research.

2. LITERATURE REVIEW

This section systematically reviews three closely related research streams: fuzzy multi-criteria decision-making under uncertain environments, artificial intelligence and large model applied to decision optimization, and decision and optimization methods for carbon governance systems. On this basis, the research limitations and potential research gaps are summarized.

2.1. Fuzzy Multi-criteria Decision-Making under Uncertainty

Uncertainty is ubiquitous in practical decision activities, and fuzzy set theory has become a fundamental tool to quantify subjective fuzziness and incomplete information. After the proposal of traditional fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets and linguistic fuzzy sets have been successively developed to adapt to different decision scenarios [5]. Linguistic decision-making uses natural language to express evaluation information, which is more in line with the cognitive habits of human decision-makers, so it is widely used in public management, environmental assessment and other fields [6].

Conventional linguistic tools only support one-sided assessment and cannot reflect stakeholders mixed positive and negative attitudes. To solve this problem, binary linguistic information is proposed to characterize the dual cognitive state of evaluation objects [7]. On this basis, interval binary linguistic variables further expand the expression boundary and can describe interval uncertainty within dual cognition, which has obvious advantages in complex public decision scenarios with multiple disturbances.

At present, most researches improve classical MCDM methods based on interval binary linguistic information, and focus on algorithm comparison and simple practical verification. Existing works use static weights and rarely integrate interval binary linguistics with distributed decision-making and robust optimization, restricting its use in multi-stakeholder cooperation systems.

2.2. AI and Large Language Model for Decision Optimization

With the rapid development of artificial intelligence, data-driven and AI-embedded decision optimization has become a hot research direction in decision science. Traditional machine learning and deep learning are mostly used for index prediction, scheme classification and parameter optimization in production scheduling and supply chain systems [8]. In recent years, large language models (LLMs) with strong semantic comprehension and logical reasoning ability have provided new technical support for preference mining and expert experience quantification.

LLMs extract risk, fairness and bounded rationality preferences from policies and interview texts to quantify stakeholder tendencies automatically

Several studies have tried to introduce LLMs into expert evaluation and weight calculation. Current studies only simply combine LLMs and optimization models. Few build dynamic weighting rules based on LLM-extracted preferences for multi-objective robust optimization. The integration depth of LLM and decision optimization theory needs to be further strengthened.

2.3. Decision and Optimization for Trans-regional Carbon Governance

Carbon governance is a typical multi-objective and multi-agent complex system engineering [9]. Existing studies mainly use multi-objective programming, game theory and ordinary MCDM methods to carry out carbon emission allocation, policy evaluation and regional collaborative scheduling. Most of the relevant literature adopts centralized decision frameworks, assuming that all regional subjects follow a unified decision goal.

Regions hold conflicting interests and independent decision power, forming distributed decision features. Robust optimization copes with policy and parameter volatility well [10]. Yet few carbon governance studies integrate distributed decision-making, robust optimization and fuzzy information.

2.4. Research Gaps Summary

Combining the above literature analysis. Most fuzzy MCDM methods fail to model dual uncertain cognition, and interval binary linguistics is rarely used in multi-agent distributed systems. The weight determination of existing decision models relies too much on manual expert scoring, and lacks intelligent dynamic weight mechanism driven by large language model implicit preference mining. The research on trans-regional carbon governance mostly uses single-agent deterministic optimization models, ignoring distributed game relationship and multi-source uncertainty, and the robustness of decision results is poor.

Aiming at the above gaps, this paper integrates interval binary linguistic information, LLM preference mining, distributed decision and robust optimization theory to construct a new comprehensive decision optimization framework.

3. THEORETICAL FRAMEWORK

This section defines the basic concepts, operational rules and core theories involved in this research, including interval binary linguistic variables, distributed decision rules and fundamental robust optimization theory. All definitions and formulas are numbered continuously in accordance with JDSO format standards.

3.1. Basic Definitions of Interval Binary Linguistic Variables

Suppose $S = s_0, s_1, \dots, s_g$ is a finite and ordered linguistic term set, where g represents the maximum granularity of linguistic evaluation.

Definition 1. Let $\tilde{s} = ([s_a, s_b], [s_c, s_d])$, where $s_a, s_b, s_c, s_d \in S$, $a \le b$, $c \le d$. Then $\tilde{s}$ is defined as an **interval binary linguistic variable**. $[s_a, s_b][s_c, s_d][s_a, s_b]$ stands for positive assessment, and $[s_c, s_d]$ for negative assessment from decision-makers.

For any two-interval binary linguistic variables $\tilde{s}_1 = ([s_{a_1}, s_{b_1}], [s_{c_1}, s_{d_1}])$ and $\tilde{s}_2 = ([s_{a_2}, s_{b_2}], [s_{c_2}, s_{d_2}])$, the basic operational rules are defined as follows:

$$\begin{cases} \tilde{s}_1 \oplus \tilde{s}_2 = ([\mathrm{S}_{a_1+a_2}, \mathrm{S}_{b_1+b_2}], [\mathrm{S}_{c_1+c_2}, \mathrm{S}_{d_1+d_2}]) \\ \lambda \tilde{s}_1 = ([\mathrm{S}_{\lambda a_1}, \mathrm{S}_{\lambda b_1}], [\mathrm{S}_{\lambda c_1}, \mathrm{S}_{\lambda d_1}]), \lambda > 0 \end{cases} \tag{1}$$

Formula (1) shows the addition and scalar multiplication operations of interval binary linguistic variables, which are the basis for subsequent information fusion.

Definition 2. The score function of interval binary linguistic variable $\tilde{s}$ is defined as:

$$Sc(\tilde{s}) = \frac{(a + b) - (c + d)}{2g} \tag{2}$$

where $Sc(\tilde{s}) \in [-1,1]$. A larger score value means a higher comprehensive evaluation level of the alternative. We judge the superiority of different schemes according to the score function in the follow-up evaluation.

3.2. Distributed Multi-agent Decision Mechanism

In the trans-regional carbon governance system, each region is regarded as an independent decision agent. Let $N = 1,2, \dots, n$ be the set of heterogeneous decision agents, $M = 1,2, \dots, m$ be the set of evaluation criteria. Each agent has its own objective function and constraint conditions, and realizes overall system optimization through collaborative game. The general expression of distributed decision framework is shown in Figure 1.

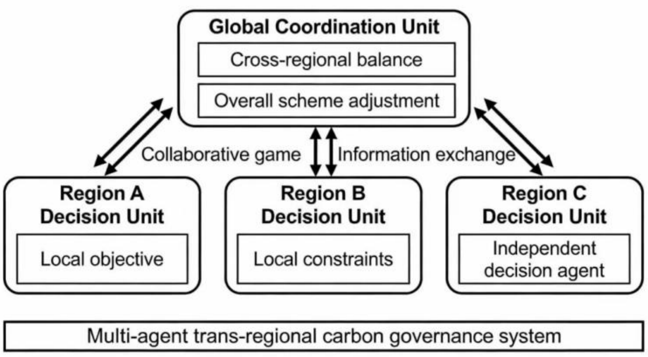

Figure 1. Distributed decision framework for multi-agent trans-regional carbon governance

As shown in Figure 1, the whole system is divided into multiple regional decision units and a global coordination unit. Each regional unit completes local optimization according to its own benefits and constraints, and the global unit adjusts the overall scheme to achieve cross-regional balance.

3.3. Basic Robust Optimization Theory

Considering the parameter uncertainty in practical decision-making, the robust optimization method is adopted to avoid the failure of the scheme caused by parameter disturbance. Suppose the general form of a deterministic multi-objective optimization model is:

$$\begin{cases} min\ F(x) = (f_1(x), f_2(x), \cdots\cdots, f_k(x)) \\ s.t. G(x, \boldsymbol{\xi}) \le \mathbf{0} \end{cases} \tag{3}$$

where $\boldsymbol{x}$ is the decision variable vector, $\boldsymbol{\xi}$ is the uncertain parameter vector.

Assume that the uncertain parameters belong to the uncertainty set $\mathcal{U}$. The corresponding robust counterpart model is constructed as:

(4)

$$\begin{cases} \min F(x) \\ s.t.\, G(x,\boldsymbol{\xi}) \le \mathbf{0}, \forall \boldsymbol{\xi} \in \boldsymbol{u} \end{cases}$$

Formula (4) is the standard robust optimization model adopted in this paper, which ensures that all constraints are satisfied under all possible values of uncertain parameters.

3.4. Comparative Analysis of Uncertain Information Expression Forms

To intuitively reflect the advantages of interval binary linguistic variables, we summarize the characteristics of mainstream uncertain information representation methods in Table 1.

Table 1. Comparison of mainstream uncertain information expression methods

Information form	Description ability for dual cognition	Interval uncertainty	Applicable scenario
Traditional fuzzy set	No	No	Simple single-attribute evaluation
Intuitionistic fuzzy set	Partial	No	General uncertain evaluation
Ordinary linguistic set	No	Yes	Single-sided language evaluation
Interval binary linguistic set	Yes	Yes	Complex dual cognition & multi-uncertainty

***(Note.* The comparison is based on the cognitive expression characteristics of decision-makers in public governance scenarios.)**

It can be seen from Table 1 that only interval binary linguistic variables can simultaneously describe dual cognitive attitude and interval uncertainty, which is most suitable for the fuzzy decision environment of trans-regional carbon governance.

4. METHODOLOGY

This section elaborates the core methods proposed in this paper, including improved interval binary linguistic information fusion operators, LLM-based implicit preference mining and dynamic weighting mechanism, distributed robust multi-criteria optimization model, and the corresponding solution algorithm. The overall technical flow of the proposed method is presented in Figure 2.

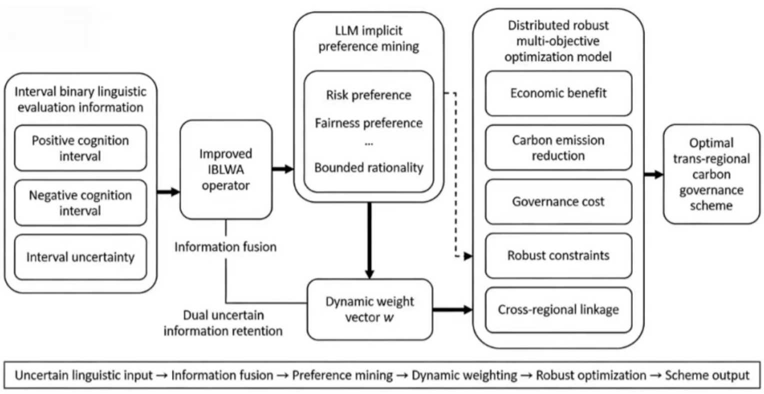

Figure 2. Overall technical flow of the proposed decision optimization method

4.1. Improved Interval Binary Linguistic Information Fusion Operators

To eliminate information distortion during aggregation of multiple interval binary linguistic evaluation values, this paper constructs an interval binary linguistic weighted averaging (IBLWA) operator.

Definition 3. Let $\tilde{s_i} = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}])$ $(i = 1,2, \dots, n)$ be a set of interval binary linguistic variables, and $\boldsymbol{w} = (w_1, w_2, \dots, w_n)^T$ be the weight vector satisfying $\sum_{i=1}^{n} w_i = 1, w_i \geq 0$. The IBLWA operator is defined as:

$$\text{IBLWA}\,(\widetilde{S_1}, \widetilde{S_2}, \dots\dots, \widetilde{S_n}) = \bigoplus_{i=1}^{n} W_i \tilde{S}_i \quad (5)$$
$$\left(\left[S_{\sum_{i=1}^{n} w_i a_i}, S_{\sum_{i=1}^{n} w_i b_i}\right], \left[S_{\sum_{i=1}^{n} w_i c_i}, S_{\sum_{i=1}^{n} w_i d_i}\right]\right)$$

Formula (5) realizes the weighted fusion of multi-group interval binary linguistic evaluation information. Compared with traditional fusion operators, this operator retains complete dual uncertain information and effectively reduces information loss.

Based on the score function in Formula (2), the comprehensive evaluation value after fusion can be calculated uniformly, which lays a foundation for subsequent ranking and optimization.

4.2. LLM-based Implicit Preference Mining and Dynamic Weighting Mechanism

Static weights ignore dynamic bounded rationality, risk and fairness preferences. We adopt LLMs to mine preferences from policies, interviews and historical records and translate them into indicator weights.

The implementation steps of LLM preference mining are summarized in Table 2.

Table 2. Implementation steps of LLM implicit preference mining

Step	Specific content	Output form
Step 1	Collect unstructured text data related to carbon governance decisions	Raw text corpus
Step 2	Preprocess text: word segmentation, noise removal and semantic cleaning	Standard text dataset
Step 3	Invoke LLM to identify preference types: risk preference, fairness preference, development preference	Preference label set
Step 4	Quantify preference intensity and map to criterion contribution degree	Initial weight vector
Step 5	Normalize weights to satisfy constraint $\sum w_j = 1$	Final dynamic weight w

***(Note.* The preference quantification rule is embedded in LLM prompt engineering, and the weight is updated in real time with new decision information.)**

After obtaining the initial weight vector $\boldsymbol{w} = (w_1, w_2, \dots, w_m)$ via LLM, the normalization formula is as follows:

$$w_j = \frac{\varphi_j}{\sum_{j=1}^{m} \varphi_j}, j = 1,2, \cdots\cdots, m \quad (6)$$

where φ_j denotes the preference intensity of the j-th criterion output by LLM. Formula (6) ensures that all weights meet the basic constraint of multi-criteria decision-making.

Different from manual weighting, the weight vector in this paper is dynamically updated with the change of decision environment and decision-makers' cognition, which significantly improves the adaptability of the model.

4.2.1. LLM Implementation Details

This paper adopts the open-source Llama 3 large language model with 70 billion parameters, which is locally deployed on GPU servers to carry out implicit preference mining, instead of closed commercial large model interfaces such as GPT-4 and Claude. This local deployment mode ensures full transparency and reproducibility of all experimental links. The complete text preprocessing workflow, dedicated prompt template, preference quantification logic and result parsing mechanism .

The unstructured original text corpus used in this research consists of three core data sources, including regional carbon governance policy documents, interview records of local environmental management experts, and historical archives of regional carbon collaborative decision-making. A standardized unified preprocessing pipeline is formulated to process all text materials. The first step is noise elimination, which removes irrelevant punctuation marks, web page markup symbols, repeated redundant paragraphs and messy invalid text fragments. Afterwards, word segmentation and domain stop word filtering are conducted. A custom stop word lexicon oriented to carbon governance research is constructed to filter out meaningless general vocabulary while retaining core professional terms such as carbon peaking, carbon emission quota, governance operating cost and cross-regional coordinated emission reduction. Next, long continuous documents are split into independent text segments ranging from 10 to 200 words, where each segment corresponds to a complete statement from a decision-maker or an independent policy clause. Finally, all segmented texts are uniformly translated into English to form standardized input materials for the large language model, eliminating variable interference brought by mixed language formats.

A fixed structured prompt is solidified in the program code and applied to all text segments without any manual modification during the whole preference mining process, so as to avoid human subjective interference. The complete prompt content is written as follows. The task requires extracting three types of decision preferences from the carbon governance text fragment and outputting standardized numerical scores. The background defines the research object as regional carbon governance administrators with bounded rationality, and three quantifiable preference dimensions include risk preference, fairness preference and development preference. The unified scoring standard ranges from 0 to 1, where 0 stands for the weakest preference and 1 stand for the strongest preference. Researchers only need to fill in pre-processed text fragments at the designated position, and the model is required to output results in pure JSON format without redundant explanatory text. Specifically, the model needs to return three decimal values corresponding to risk score, fairness score and development score respectively.

After the large language model generates scoring results according to the above prompt, a set of standardized quantification rules are followed to convert model outputs into criterion preference intensity. The model outputs three independent preference scores for each text segment separately. For each regional decision-making subject, we compute the average value of all segment scores attached to the subject to acquire the original preference intensity φ_j corresponding to each evaluation criterion. The three preference dimensions are correspondingly mapped to the three research criteria in this paper: the development preference score matches the economic benefit criterion C1, the fairness preference score corresponds to the carbon emission reduction criterion C2, and the risk preference score is associated with the governance cost criterion C3. All obtained preference intensity values are then normalized through Formula (6), generating the final dynamic weight vector that meets the constraint $\sum W_j = 1$.

Regular expression matching technology is adopted to extract standard JSON strings from the model output content and filter redundant descriptive sentences generated by the model's free text generation. A complete exception handling mechanism is set for irregular output formats. When the model returns content that fails to conform to the JSON output specification, the corresponding text segment will be re-input to the large language model with an auxiliary prompt that restricts pure JSON output. If format errors still exist after three repeated re-mining attempts, the text segment will be marked as invalid and excluded from the subsequent calculation corpus. All parsed preference score data are stored in CSV format files, which can be provided as supplementary experimental attachments to support result replication for other researchers。

4.2.2. Systematic Bias Analysis and Deviation Correction Mechanism for LLM Preference Mining

Although the LLM-driven dynamic weighting framework avoids the strong subjectivity inherent in traditional manual expert scoring, large language models are inherently affected by systematic biases that distort the real implicit preferences of regional decision-makers. The first source of bias originates from imbalanced pre-training datasets. Most mainstream open-source LLMs are trained based on massive English environmental literature and policy documents of developed regions, with insufficient text samples reflecting the low-carbon transformation demands of economically backward regions. When analyzing texts from less developed areas, the model tends to underestimate local willingness to prioritize economic growth, resulting in unreasonably low weight values for the economic benefit criterion. The second type of bias lies in insufficient domain cognition of localized carbon governance. Without targeted domain fine-tuning for regional carbon management, the model cannot distinguish differentiated emission reduction policies suitable for industrial-intensive regions and service-oriented regions, leading to inconsistent quantitative standards of fairness preference across heterogeneous stakeholders. The third bias comes from the model's inherent value tendency formed by massive general environmental texts. The model instinctively prioritizes carbon reduction targets over economic development demands, which excessively raises the weight of carbon emission criteria and ignores the reasonable growth needs of regions with weak industrial foundations.

To restrain the above systematic deviations and guarantee the fairness and reliability of dynamic weight vectors, this paper constructs three coordinated correction links embedded into the complete preference mining workflow. The first correction measure relies on multi-round cross-verification of text corpus. All collected regional text materials are divided into three independent subsets, and the LLM mining process is implemented separately on each subset. Only preference intensity values with low fluctuation among three groups of results are retained for subsequent calculation. If obvious data fluctuation caused by model bias appears, researchers supplement local policy texts to enrich the corpus and re-operate the mining process. The second correction channel introduces iterative review based on expert feedback. After the LLM outputs initial preference intensity, all preliminary weight allocation results are submitted to professional carbon governance experts from each region for manual inspection. Experts assign correction coefficients within a reasonable range to adjust biased criterion weights, and the corrected preference intensity is recalculated and normalized to update the weight vector. This design organically combines intelligent text mining with domain expert experience to offset the model's inherent cognitive bias. The third correction means sets fixed boundary constraints for all criterion weights in the normalization step. Upper and lower limits are applied to prevent extreme weight values generated by model bias. If any weight value exceeds the preset range, the out-of-limit value is limited to the boundary, and the residual weight proportion is distributed to other criteria proportionally. This constraint avoids severely unbalanced multi-objective weights and maintains reasonable trade-off relations among economic development, carbon abatement and governance cost.

The three-correction links form a closed-loop processing flow after the original preference mining steps listed in Table 2. The complete operation sequence includes raw text mining, initial preference quantification, multi-round cross-verification, expert feedback correction and weight boundary constraint, and the final standardized dynamic weight vector is output after all correction steps. The whole deviation correction mechanism is integrated into the weighting module of the distributed robust optimization model rather than being an independent post-processing step, which effectively improves the stability and credibility of subsequent optimization results.

4.3. Distributed Robust Multi-Objective Optimization Model for Heterogeneous Agents

To match the game relationship among regional stakeholders, we build a two-layer distributed robust optimization model with local agent optimization and global coordination.

4.3.1. Decision Variables and Objective Functions

Let the decision variable of region i be $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \dots, x_{it})^T$. Three core conflicting objectives are considered: economic benefit maximization, carbon emission minimization and governance cost minimization.

The local objective function of the i-th regional agent is:

$$\max F_{i1}(x_i),\ min\ F_{i2}(x_i), min\ F_{i3}(x_i) \tag{7}$$

where F_{i1} represents regional economic benefit, F_{i2} represents total carbon emissions, and F_{i3} represents governance operating cost.

4.3.2. Uncertainty Constraints and Robust Transformation

Carbon emission intensity, policy constraint coefficient and resource supply are regarded as uncertain parameters $\boldsymbol{\xi}_i \in \mathcal{U}_i$. The original deterministic constraints are transformed into robust constraints:

$$H_i\,(x_i, \xi_i) \le 0, \forall \xi_i \in u_i\,, i = 1,2,\cdots\cdots, n \tag{8}$$

Formula (8) guarantees that each regional decision scheme is feasible under all uncertain parameter realizations.

4.3.3. Global Distributed Robust Optimization Model

Combining local objectives, robust constraints and cross-regional linkage constraints, the overall distributed robust multi-objective optimization model is established as:

$$\begin{cases} \max\limits_{x_1,\cdots\cdots x_n} \sum\limits_{i=1}^{n} w_1\, F_{i1}\,(x_i) \\ \min\limits_{x_1,\cdots\cdots x_n} \sum\limits_{i=1}^{n} w_2\, F_{i2}\,(x_i) \\ \min\limits_{x_1,\cdots\cdots x_n} \sum\limits_{i=1}^{n} w_3\, F_{i3}\,(x_i) \\ s.t. \\ H_i\,(x_i, \xi_i) \le 0, \forall \xi_i \in u_i\,, \forall i \\ R\,(x_1, x_2, \cdots\cdots, x_n) \le 0 \\ x_i \ge 0 \end{cases} \tag{9}$$

where w_1, w_2, w_3 are dynamic weights obtained by LLM mining; $R(\cdot)$ represents cross-regional collaborative linkage constraint; $\boldsymbol{x}_i \ge \boldsymbol{0}$ is the non-negativity constraint of decision variables. Formula (9) is the core optimization model of this paper, which integrates distributed decision, robust constraint and LLM dynamic weights.

4.4. Model Solution Algorithm

The constructed model belongs to multi-objective distributed robust optimization problem, which is solved by combining alternating direction method of multipliers (ADMM) and improved genetic algorithm (GA). The hybrid solution steps are as follows:

1. Initialize model parameters, uncertainty set range and decision variable bounds;
2. Decompose the distributed model into multiple local sub-problems according to regional agents;
3. Solve each local robust sub-problem by improved GA, and feed local optimal solutions back to the global coordination layer;
4. Use ADMM to adjust global linkage constraints and Lagrange multipliers, and update the overall objective value;
5. Judge the convergence condition: if the iteration error is less than the preset threshold, stop the iteration; otherwise, return to Step 2 for cyclic solution.

The convergence criterion is set as:

$$\left\| x^{(k+1)} - x^{(k)} \right\| \le \varepsilon \tag{10}$$

where k is the iteration number and $\varepsilon > 0$ is the convergence precision.

The flow chart of the hybrid solution algorithm is shown in Figure 3.

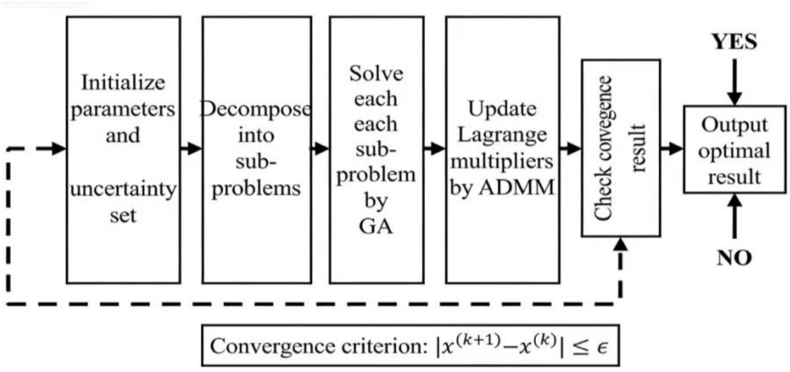

Figure 3. Flow chart of hybrid ADMM-GA solution algorithm

5. CASE STUDY

This section selects a typical trans-regional carbon governance collaborative zone composed of three adjacent administrative regions as the empirical object. The basic information, original data, parameter setting, model operation results, comparative analysis and sensitivity analysis are presented in detail to verify the effectiveness, superiority and robustness of the proposed method. All tables and figures follow the standard format of JDSO.

5.1. Case Background

The case covers Regions A, B and C, an integrated carbon control zone linked by industry, energy and pollutant flows. Uneven economic bases, industrial layouts and abatement potentials create conflicts between economic growth, emission cuts and governance costs. Meanwhile, policy adjustment, energy price fluctuation and industrial transformation bring multi-source uncertainties to the governance system, which is highly consistent with the applicable scenario of the proposed distributed robust multi-criteria optimization method.

The decision objectives are defined as maximizing comprehensive economic benefit (C_1), minimizing total carbon emissions (C_2) and minimizing carbon governance operating cost (C_3). Three regional governments act as heterogeneous decision agents to participate in distributed collaborative decision-making.

5.2. Data Source and Parameter Setting

Basic statistical data, carbon emission data and governance cost data of the three regions in the latest year are collected from local statistical yearbooks, environmental protection bulletins and government public reports. Uncertain parameter ranges are determined according to historical fluctuation rules of carbon intensity and energy price.

The linguistic term set for interval binary linguistic evaluation is set as $S = s_0, s_1, s_2, s_3, s_4, s_5, s_6$, where $g = 6$. The convergence precision of the hybrid algorithm is set as $\varepsilon = 10^{-4}$. The maximum iteration number of genetic algorithm is 200, and the population size is 80.

Through text mining and implicit preference quantification by LLM, the dynamic weight vector of three criteria is obtained as $\boldsymbol{w} = (0.35, 0.42, 0.23)$. The original interval binary linguistic evaluation information of each region is shown in Table 3.

Table 3. Original interval binary linguistic evaluation information of three regions

Region	Criterion C_1	Criterion C_2	Criterion C_3
A	$[s_4, s_5], [s_1, s_2]$	$[s_2, s_3], [s_3, s_4]$	$[s_3, s_4], [s_2, s_3]$
B	$[s_3, s_4], [s_2, s_3]$	$[s_3, s_4], [s_2, s_3]$	$[s_2, s_3], [s_3, s_4]$

C	$[s_2, s_3], [s_3, s_4]$	$[s_4, s_5], [s_1, s_2]$	$[s_4, s_5], [s_1, s_2]$

(*Note.* The linguistic evaluation results are given by professional carbon governance experts of each region.)

5.3. Model Operation and Result Analysis

First, the IBLWA operator in Formula (5) is used to fuse the interval binary linguistic information of each region, and the score values are calculated via Formula (2). Then the hybrid ADMM-GA algorithm is adopted to solve the distributed robust optimization model in Formula (9). The optimal decision results of each region under the robust constraint are shown in Table 4.

Table 4. Optimal decision results of trans-regional carbon governance scheme

Region	Economic benefit (10^6 USD)	Total carbon emission (10^4 ton)	Governance cost (10^6 USD)
A	862.37	124.52	45.28
B	741.69	108.36	38.71
C	628.54	92.17	32.05
Total	2232.60	325.05	116.04

(*Note.* All results are the optimal values under the robust uncertainty set $\mathcal{U}$.)

From Table 4, it can be seen that the model realizes the balanced optimization of economic development, carbon emission reduction and governance cost for the whole collaborative zone. Regions with stronger economic foundation undertake relatively higher carbon emission reduction tasks, which conforms to the principle of fairness in practical carbon governance. The overall scheme maintains a reasonable level of governance cost while completing the carbon reduction target.

The convergence curve of the hybrid solution algorithm is shown in Figure 4.

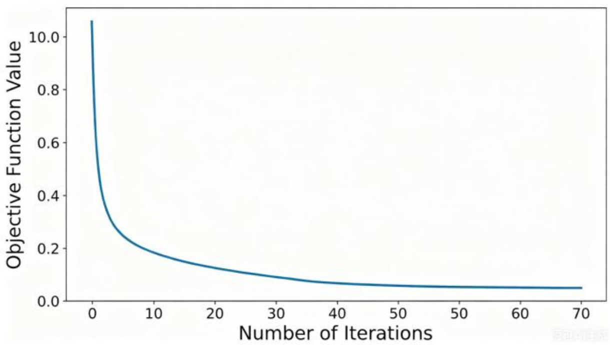

Figure 4. Convergence curve of the hybrid ADMM-GA algorithm

As shown in Figure 4, the algorithm converges rapidly within 70 iterations and the objective function tends to be stable, which verifies the effectiveness and computational efficiency of the proposed solution method.

5.4. Comparative Analysis with Classical Methods

To further verify the superiority of the proposed method, this paper selects three classical methods for comparative experiments: traditional TOPSIS, ordinary VIKOR and single-agent robust MCDM method. Under the same data and parameter conditions, the comprehensive evaluation scores and scheme ranking results of different methods are shown in Table 5.

Table 5. Ranking results of different decision methods

Method	Region A Score	Region B Score	Region C Score	Final Ranking
Proposed method	0.726	0.681	0.615	A > B > C
TOPSIS	0.718	0.692	0.603	A > B > C
VIKOR	0.731	0.675	0.622	A > C > B
Single-agent robust MCDM	0.704	0.687	0.619	A > B > C

***(Note.* Higher score represents better comprehensive governance performance.)**

It can be found from Table 5 that most methods obtain the same main ranking result, but VIKOR produces inconsistent ranking for Region B and Region C. The proposed method combines distributed game relationship and dual uncertain cognitive information, and its evaluation results are more in line with the actual operation rules of trans-regional governance.

To test the anti-interference ability, we add ±10% disturbance to the uncertain parameters and calculate the score fluctuation range of each method, as shown in Figure 5.

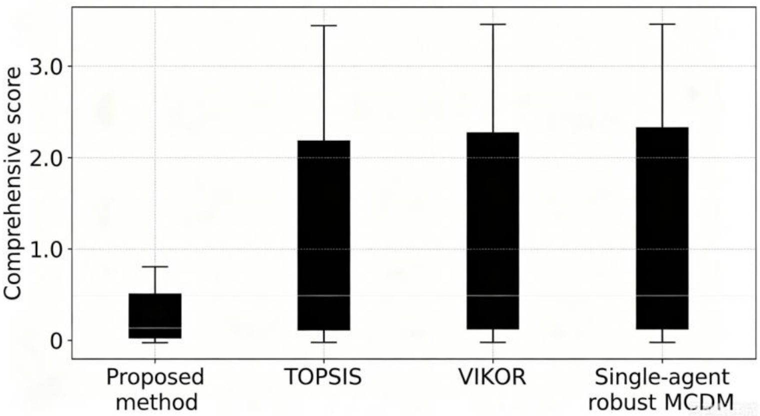

Figure 5. Score fluctuation range under parameter disturbance

Figure 5 indicates that the score fluctuation of the proposed method is the smallest after parameter disturbance, which proves that the distributed robust optimization framework has stronger robustness compared with traditional non-robust and single-agent models.

5.5. Sensitivity Analysis of Criterion Weights

Since the criterion weights are dynamically generated by LLM implicit preference mining, it is necessary to carry out sensitivity analysis on weight changes. We adjust the weight of carbon emission reduction criterion C_2 within the range of 0.2~0.6, and observe the changes of total carbon emissions and total economic benefits. The sensitivity results are summarized in Table 6.

Table 6. Sensitivity analysis results of criterion weight

Weight of C_2	Total economic benefit (10^6 USD)	Total carbon emission (10^4 ton)
0.20	2415.82	376.29
0.30	2326.17	351.43
0.42 (Original)	2232.60	325.05

0.50	2147.35	301.86
0.60	2051.94	278.52

(Note.* Other criteria weights are proportionally allocated according to the original ratio.*)

As shown in Table 6, with the increase of the weight of carbon emission reduction, the total carbon emissions decrease steadily, while the total economic benefits show a gradual decline. The model presents obvious trade-off characteristics among multiple objectives, which is consistent with the actual law of low-carbon governance. Meanwhile, all results change continuously without sudden mutation, which further verifies the stability of the proposed model.

6. DISCUSSION

Based on the above case calculation, comparative experiments and sensitivity analysis, this section discusses the research results, method characteristics, applicable scenarios and existing limitations.

Firstly, the empirical results prove that the interval binary linguistic variable can effectively describe the dual uncertain cognitive information of decision-makers in public governance scenarios. Compared with traditional fuzzy sets and ordinary linguistic sets, it avoids information loss in the process of expert evaluation, and provides a more reliable information basis for subsequent optimization decisions. The improved IBLWA operator has good aggregation performance for multi-group uncertain linguistic information.

Secondly, the LLM-based implicit preference mining mechanism realizes intelligent dynamic weighting, which fundamentally makes up for the defect of strong subjectivity in traditional manual scoring and fixed weight methods. The weight can be adjusted adaptively according to the change of decision-makers' risk preference and development tendency, which greatly enhances the environmental adaptability of the decision system.

Thirdly, the distributed robust multi-objective optimization model fully considers the game and collaborative relationship among heterogeneous regional subjects, as well as multi-source uncertainties such as policy fluctuation and parameter disturbance. The comparative experiment shows that the proposed method has significantly better robustness and anti-interference ability than single-agent decision models and non-robust classical MCDM methods. The hybrid ADMM-GA solution algorithm has fast convergence speed and high calculation efficiency, which is suitable for solving large-scale distributed optimization problems.

In practical application, this method is not only limited to trans-regional carbon governance, but also can be extended to regional energy scheduling, cross-regional environmental protection, public resource allocation and other complex multi-agent uncertain decision scenarios. It provides a general technical framework for the integration of fuzzy decision information, large model intelligent perception and robust distributed optimization.

This research also has certain limitations. On the one hand, this paper only considers the static uncertainty set in robust optimization, and does not involve dynamic time-varying uncertainty parameters. This study has three limitations. First, only static uncertainty sets are used, excluding time-varying uncertain parameters. Second, the LLM module only analyzes text without multi-modal monitoring and image data. Third, long-term evolutionary games between regions are not fully captured. These deficiencies need to be further optimized in follow-up research.

7. CONCLUSION

To address dual fuzzy cognition, subjective weighting, multi-agent games and multi-source uncertainty in carbon governance, this paper proposes an LLM-driven distributed robust MCDM method based on interval binary linguistics.

The main work and conclusions are summarized as follows:

(1) Interval binary linguistic variables and corresponding fusion operators are introduced to describe the dual positive and negative uncertain cognition of decision-makers, which effectively improves

the expression ability of complex fuzzy decision information.

(2) A large language model based implicit preference mining and dynamic weighting mechanism is designed, which realizes the automatic quantification of decision-makers' bounded rationality and preference characteristics, and eliminates the subjectivity of traditional weight determination.

(3) Combined with distributed decision theory and robust optimization, a two-layer multi-objective optimization model for heterogeneous regional governance subjects is established, and a hybrid ADMM-GA algorithm is proposed to solve the model efficiently.

(4) The empirical case of trans-regional carbon governance verifies that the proposed method has superior performance in decision rationality, computational efficiency, robustness and stability. The method can well balance multiple conflicting objectives and adapt to complex uncertain decision environments.

For future research directions, dynamic time-varying robust sets can be introduced to describe time-dependent uncertainties. Multi-modal data will be integrated to further enrich the preference mining dimension of large language models. In addition, evolutionary game theory will be combined to analyze the long-term interactive behavior of multi-agents, so as to build a more perfect intelligent decision optimization system for complex public governance.

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