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ARTICLE

Profit Maximization in Unbalanced Fuzzy Transportation Networks Using Trapezoidal Fuzzy Modeling and Exact Optimization

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Abstract

Profit-oriented logistics systems operate under considerable uncertainty in profit margins, supply capacities, and demand levels, which challenges traditional transportation modeling assumptions. However, a large portion of the transportation literature still relies on deterministic formulations or simplified fuzzy representations that are not sufficient to reflect parameter stability. Although many fuzzy transportation models have been developed using triangular fuzzy numbers, they are often coupled with heuristic solution methods, which may oversimplify real operational conditions and provide limited insight into the stability of resulting decisions. Consequently, the robustness of such models as practical logistics planning tools remains limited. This paper proposes a solver-based optimization model for profit maximization in unbalanced transportation networks under uncertainty. Trapezoidal fuzzy numbers are used to represent imprecise profits, supplies, and demands, explicitly capturing stable operational ranges under both optimistic and pessimistic scenarios. A structured defuzzification approach is employed to compute decision-ready parameters, while the resulting linear programming model guarantees globally optimal solutions. The proposed framework is demonstrated through an applied case study based on the Egyptian petroleum distribution network. The results show that the model produces consistent profit outcomes and stable shipment decisions across different uncertainty levels and defuzzification rules. Furthermore, sensitivity and robustness analyses confirm that the dominant allocation patterns remain stable under moderate variations in the uncertainty structure, supporting the practical relevance of the proposed approach as an effective decision-support tool for transportation planning.

Keywords: Fuzzy transportation problem; Profit maximization; Trapezoidal fuzzy numbers; Unbalanced transportation networks

1. INTRODUCTION

Transportation networks are an integral part of contemporary supply chains, significantly determining operational performance and economic activity. In systems that are extremely logistics-oriented, the transfer of goods from factories, warehouses, and sales points is not a support activity but a considerable driver of profit. Transport decisions impact lead times, resource management, and cost structures, with their cumulative effects influencing the competitive edge in various industries[1, 2].

In commercial logistics systems, transport planning ceases to be a mere cost reduction tool, becoming a strategic means of increasing profit. The allocation of shipments, routing, and utilization of capacity determines which resources are materialized into economic value. These choices are usually made in far from ideal circumstances. Margins fluctuate due to inconsistent fuel costs, varying operational rates, and unpredictable demand. Meanwhile, supply potentials and demand levels can also be disrupted by maintenance outages, regulatory constraints, seasonal patterns, and erratic consumption behavior. From

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this, it follows that the parameters specifying transportation problems in practical applications are unlikely to be fixed and may not even be accurately known over short planning periods[3, 4].

Despite this operational aspect, many classical models in transportation are based on some deterministic assumptions that consider profits, supplies, and demands as precise quantities. Although these models provide mathematically tractable problems, they do not account for the stochasticity in practical logistics systems and can sometimes yield systems of solutions that are sensitively dependent on parameter variations. To alleviate this drawback, uncertainty-aware modeling methods have been proposed, and fuzzy sets are among the most prominent ones. However, much of the literature on fuzzy transportation employs simple approximations based on the underlying assumption that a single most likely value can be surrounded by uncertainty in this manner. This is not a satisfactory interpretation of systems in which systemic parameters have the same value over a finite range, rather than at a single point.[5]

Aside from their representational shortcomings, the solving methods used in most models are another factor that restricts the applicability of these models. Heuristic and semi-manual solution procedures are often applied (especially in fuzzy transportation problems), which do not guarantee optimal solutions and make it difficult to distinguish between the genuine effects of modeling and the artifacts of a solution. Furthermore, robustness is often ignored or implicit at best, with the reported results limited to only a few isolated cases, as opposed to spanning across uncertainties. The absence of a systematic evaluation of robustness also undermines one's confidence in using such models as decision-support instruments[6, 7].

These limitations motivate the development of an advanced transportation modeling framework that explicitly considers uncertainty, without compromising analytical rigor or numerical solution quality. This paper aims to address this problem by establishing a profit-maximization transportation model with uncertainty using an extended parameter set and solving the model with an exact optimization approach. The framework is designed to preserve the actual structure of practical logistics systems, avoid heuristic procedures, and make decision stability measures under uncertainty explicit.

The contributions of this study are threefold. First, a profit-based transport model that considers the revenue/risk of profits, and explicitly incorporates uncertain profits, production capacities, and demands, is presented. Second, trapezoidal representation is applied to reflect the stability spectrum that can frequently be encountered in logistics activities, thereby enhancing the existence of generalized uncertainty modeling. It also uses only exact linear programming solvers for global optimality and supplements the analysis with sensitivity and robustness studies. The methodological value of these contributions is realized through a realistic case study grounded in logistics and fuel distribution networks in Egypt, demonstrating the real-world relevance of the developed methodology.

2. Literature Review

2.1. Classical Transportation and Profit Models

The transportation problem is one of the earliest and classic applications of linear programming in operations research. The most typical formulation of this problem represents the distribution of shipments from multiple supply points to various demand nodes, subject to restrictions on capacity availability and consumer demand, with the primary objective being the minimization of total transportation cost. This framework is well synthesized in Kacher et al. [8], which presents a historical overview of the transportation problem and discusses its mathematical model on balanced and unbalanced networks. They stress that unbalanced networks naturally occur, stemming from forecast errors, differing demands across seasons, and operational interruptions, rather than being extreme modeling instances.

Hence, the solution to unbalanced transportation networks has received continued attention. Rashid et al. [9] solve this problem by developing systematic methods for generating dummy node insertions. The most important innovation of their work lies in the rejection of the commonly held view that dummy allocations incur no cost at all, thereby neglecting the economic implications of excess capacity or unsatisfied demand, which may lead to managerial myopia. With non-zero dummy costs, their method remains economically meaningful and computationally feasible.

Although cost reduction has been a primary focus in previous transportation literature, it appears that recent efficiencies are not easily achieved without a different mindset of efficiency. Bhattarai et al. [10] explicitly recast the transportation problem as a profit-maximization model and showcased via sectoral applications that the logistics decisions are becoming more and more pervaded by the revenue component, rather than merely focusing on cost. This transformation in modeling viewpoint establishes a crucial conceptual foundation for business-oriented transportation models, encouraging the consideration of extensions such as uncertain demand and decision robustness.

2.2. Fuzzy Transportation Models

Due to the limitations of deterministic transportation models, fuzzy set theory has been extensively used as a method for characterizing uncertainty in transportation parameters. A cornerstone in this direction is established by Kaur et al. [11], who present a complete transportation problem considering both unit cost, capacity, and demand as fuzzy numbers. Their own contribution has shown that partial fuzzification of parameters entails conceptual conflicts, so that only complete fuzziness provides an adequate representation of decision inputs[12].

However, a large part of the following publications north of the line continue to use triangular fuzzy numbers, as these are analytically simple. However, it turns out that the triangular representations are inherently based on a single most likely value for parameters, and this might not correspond to logistical reality, because in practice, parameters tend to be stationary over periods. The use of trapezoidal fuzzy numbers has been introduced as a more realistic alternative. Deshmukh et al. [13] establish ranking methods specialized for trapezoidal fuzzy transportation problems with unbalanced networks, focusing on the fact that, instead of collecting point estimates, it is more suitable to acquire stability regions. Nathiya et al. [14] continue this trend by proposing an algorithmic approach to Robust ranking, which strikes a balance between computational efficiency and interpretability for trapezoidal fuzzy parameters in robust optimization.

Some of the other studies aim to utilise the fuzzy information from start to end, and not necessarily by using early defuzzification. Niksirat et al. [15] suggest a three-phase method that directly evaluates feasibility and optimality in the fuzzy space before producing crisp decisions, thereby maintaining uncertainty information at a more stagematic level of hierarchy. Extensions for richer fuzzy representations have been further studied. Bishtet al. [16] develop pentagonal and hexagonal fuzzy numbers and illustrate their capability to capture complex patterns of uncertainty at the cost of more complex modelling and computation.

In addition to these exact or semi-exact methods, heuristic techniques continue to be popular. Kacher et al. An improved approximation method for balancing fuzzy requirement transportation problems without using dummy nodes is presented in [17], which works directly with triangular fuzzy data. Although heuristics can often lead to faster computation, they usually fail to ensure optimality, which is an ongoing compromise between computational speed and solution quality.

2.3. Defuzzification and Ranking Techniques

It is essential to note that defuzzification is a crucial epistemic step in operationalising fuzzy transportation models and involves transforming fuzzy parameters into crisp ones, on which optimisation can be carried out. Most of the popular ranking methods are centroid-based and mean-based, which have been recommended for their simplicity and interpretability. Nishad et al. [18] provide a systematic comparison of several defuzzification families, demonstrating that the optimization performance may vary significantly when different ranking techniques are applied to the same fuzzy inputs. Their investigation highlights that defuzzification is not a neutral preprocessing step, but rather it is a modeling decision with a direct impact on the quality of the solution from a solution perspective.

Alternative paradigms are now challenging defuzzification-based methodologies. Rishabh et al. [19] design a decomposed fuzzy decision-making model and combine it with metaheuristics to preserve fuzzy knowledge throughout the solution computation, thereby minimizing information loss. Bharati et al. [20] generalized transportation problems for interval-valued intuitionistic fuzzy sets, enabling the inclusion of both membership and nonmembership information, and appropriately amended the standard transportation algorithms. Work conducted in this context addresses multi-objective fuzzy transportation formulations [21], heuristic modifications based on knapsack problem structures [22], and efficiency-oriented optimization involving Fermat fuzzy sets [23]. Together, these contributions demonstrate the range of possible rankings and solutions, while highlighting a growing methodological complexity.

2.4. Identified Research Gap

Recent applications of decision stability and robustness in transportation- and logistics-related optimization underscore the increasing relevance of the concept. Bilişik et al. [24] demonstrate, using interval-valued intuitionistic fuzzy-based CRITIC–TOPSIS analysis, that the perception of sensitivity can significantly influence mode choice and argue for evaluating solution robustness in different models. This view is supported by more comprehensive research on the supply chain. Alinezhad et al. [25] and Mirzagoltabar et al. [26] demonstrate that integrating fuzzy modeling and exact optimization, as well as robustness analysis, is possible and reasonable in complex logistics and closed-loop supply chain systems. In addition to fuzzy-based approaches, Luo et al. [27] present a stochastic–robust optimization approach and demonstrate that different paradigms for treating uncertainty have specific strengths and weaknesses.

Collectively, these studies have led to an apparent deficiency in the literature. However, few works consider maximization with interval profit, trapezoidal fuzzy data, and solve the exact optimization problem using a solver for the systematic robustness analysis of these problems in the context of a realistic case study, such as logistics. Classical methods do not capture uncertainty; fuzzy techniques employ simplifying triangular forms or heuristic solvers, and robustness is generally treated implicitly. To address this gap, we present a general framework developed in this study, which integrates realistic uncertainty modeling with exact optimization and explicit robustness assessment.

3. Fuzzy Modeling Preliminaries

This section presents the basics of fuzzy modeling used in this work and develops a numerical language needed for the optimization framework. The trapezoidal fuzzy numbers are utilized to describe the uncertainty of transport profits, supply capacities, and demand requirements. The section aims only to provide a sketch of definitions and interpretations, leaving forms of defuzzification (and numerical transformations) for later sections to avoid blurring concepts.

3.1. Trapezoidal Fuzzy Numbers

In this investigation, uncertain parameters are modeled using a Trapezoidal Fuzzy Number, which captures the fact that the parameter has a range of stable operation, instead of just one most likely value. Mathematically, a trapezoidal fuzzy number $\tilde{A}$ Four real numbers give it, and that is defined as

$$\tilde{A} = (a, b, c, d), \quad a \le b \le c \le d,$$

Where the parameters a and d represent the lower and upper bounds of the fuzzy set, respectively, while b and c define the interval in which the membership degree reaches its maximum value.

The membership function associated with the trapezoidal fuzzy number $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \le x < b, \\ 1, & b \le x \le c, \\ \dfrac{d-x}{d-c}, & c < x \le d, \\ 0, & x > d. \end{cases}$$

In comparison with triangular fuzzy numbers, which indicate a unique representative value, the trapezoidal definition includes a plateau region $[b, c]$. This range can be described as a region of stability – the value of the parameter is regarded as entirely plausible and independent of small changes. From a pragmatic point of view, this property is particularly suitable in the transportation context, where profits, capacities, and demands can be considered relatively constant over a planning horizon. At the same time, only scenarios at the extremes (captured by the bounds a and d) are affected by uncertainty.

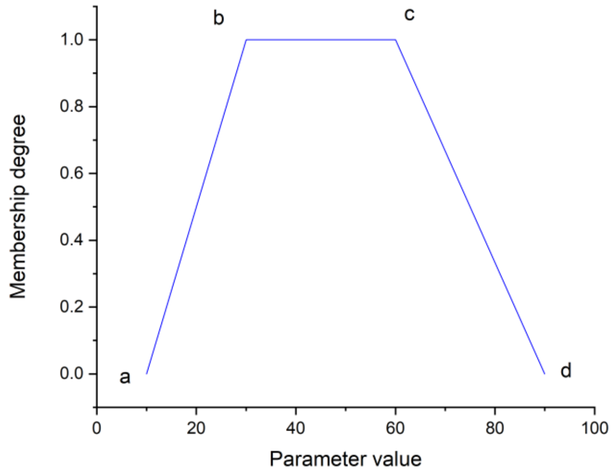

Figure 1: Geometric representation of a trapezoidal fuzzy number

A geometric representation of a trapezoidal fuzzy number is shown in Figure 1, which presents an epitome for the membership function and emphasizes the importance of its plateau region. This example illustrates the distinctions between trapezoidal and triangular representations, as well as the modeling flexibility afforded by the stability interval.

3.2. Fuzzy Representation of Transportation Parameters

In practice, transportation and logistics networks have some important parameters that are intrinsically uncertain but not deterministic. In profit-maximizing transport situations, unit profits depend on several fluctuating factors, including fuel prices at the time of operation, general running costs, market demand characteristics, and contract-bound price structures. These considerations result in profit margins that are not fixed, but instead fall within a range of uncertainty, for which a fuzzy representation is more suitable than a point estimate.

Transportation supply capacity is also uncertain due to operational disturbances, work schedule maintenance, labor availability, and regulatory limitations. While nominal capacity has a definite value, deliverable capacity typically fluctuates within a defined range that encompasses an intermediate inclusive region where regular operation occurs. The demand component behaves similarly, fluctuating around a steady range rather than an exact amount, influenced by seasonal, economic, and consumer behavior effects.

Trapezoidal fuzzy numbers are a natural and intuitive approach to modeling this kind of feature. The lower bound a characterizes pessimistic situations (e.g., low demand or little capacity for performing work), the upper bound d reflects optimistic ones. The plateau area of the region $[b, c]$ Corresponds to normal operating conditions in which the parameter is most likely to occur and remains relatively stable. By using this representation, the early oversimplified models would be able to retain crucial information about uncertainty in the model.

No defuzzification or numerical conversion is performed at this stage. The fuzzy parameters are preserved in their pure trapezoidal form to explicitly model the corresponding uncertainty, which will be further introduced into the objective and constraints within the optimization framework step-by-step.

4. Defuzzification and Robustness Metrics

This section describes the procedures for transforming fuzzy uncertainty into operational decision values, preserving important information linked to variability and stability. Through the proposed defuzzification rules and robustness measures, this method, rather than suppressing early uncertainty, facilitates an explicit investigation of how uncertainty affects optimal decisions. Two defuzzification schemes are employed: a centroid-based approach, which serves as the primary transformation process, and a mean-based approach, which is disregarded here for benchmarking purposes only. Moreover, quantitative last-stage criteria are proposed to measure the strength and stability of the final iteration results.

4.1. Centroid-Based Defuzzification

In this study, the centroid algorithm is employed as the primary defuzzification process to convert trapezoidal fuzzy numbers into real-valued quantities in optimization problems. The centroid method is considered one of the most interpretable defuzzification methods, as it directly captures the concept of a geometric center for the fuzzy set, accounting for all parts of the membership function rather than a single representative point.

From a decision-making perspective, it is evident that the centroid is based on a fair combination of pessimistic and optimistic cases. The lower and upper bounds, which are the two segments of a trapezoidal fuzzy number, determine the final value, but a greater weight is placed on the plateau (middle region) that represents a stable and complete plausible operation, irrespective of any of its off-starts, as a rule. This weighting scheme is highly suitable for transportation, as decisions are frequently based on typical, non-extreme operating ranges but need to be robust to inversions in the normal or favorable range.

The centroid defuzzification method is adopted due to its balanced treatment of the entire support of trapezoidal fuzzy numbers, as it accounts for both the core and the spread of uncertainty, making it particularly suitable for transportation planning problems where stability of decisions is required.

For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, the centroid-based defuzzified value is computed as:

$$R_C(\tilde{A}) = \frac{a + 2b + 2c + d}{6}.$$

As shown in the equation, the parameters b and c associated with the stability region receive more substantial influence, while the extreme bounds a and d moderate the result by reflecting uncertainty at the margins. This property ensures that the defuzzified value remains grounded in realistic operating conditions without disregarding uncertainty.

4.2. Mean-Based Defuzzification (Benchmark)

Aside from the centroid-based method, a mean-based approach for defuzzification is also considered for comparison. This representation calculates the normal average of the four defining parameters for a trapezoidal fuzzy number; moreover, all factors are handled symmetrically. Although it is simple and computationally easy, the mean-based method has no specific preference for stable over marginal regions of the fuzzy set.

The mean-based defuzzified value of $\tilde{A} = (a, b, c, d)$ is given by

$$R_M(\tilde{A}) = \frac{a + b + c + d}{4}.$$

It should be clear that the mean-based method is not advertised as more reasonable or realistic than the centroid approach. It does not become a static model, but rather a potential tool to assess how the alternative defuzzification rule impacts the explicit computation of optimal shipment decisions and profit values. Any discrepancies between the solution with two equations for the transportation model inform us about the impact of how we handled uncertainty representation.

4.3. Robustness and Stability Metrics

Defuzzification is insufficient for assessing the quality of decisions in uncertain environments. As a result, more "robustness and stability" indices are proposed to measure how uncertainty influences not only parameters but also optimal solutions.

The total width of uncertainty for a trapezoidal fuzzy number is considered as the fuzziness width, which can be expressed as

$$W = d - a.$$

This metric captures the total range of possible variation and indicates the level of uncertainty surrounding a parameter.

Complementary to this measure, the plateau width is defined as

$$P = c - b.$$

The plateau width corresponds to the extent of the stability region, where it is deemed that the parameter is entirely viable. Higher P shows a broader zone of stability and corresponds to a more stable decision environment.

To evaluate the consistency of the most fitting shipment strategy between diverging defuzzification schemes or different sources of uncertainty, a Decision Stability Index (DSI) was proposed. Let $x_{ij}^{(1)}$ and $x_{ij}^{(2)}$ Denote the optimal shipment quantities obtained under two different modeling conditions. The DSI is defined as

$$DSI = 1 - \frac{\#\left\{(i,j): x_{ij}^{(1)} \neq x_{ij}^{(2)}\right\}}{mn},$$

Where m and n denote the number of supply and demand nodes, respectively, the DSI takes values between zero and one, with higher values indicating greater consistency between solutions. A DSI close to unity suggests that optimal decisions are insensitive primarily to modeling choices or variations in uncertainty, thereby indicating strong robustness.

5. Mathematical Optimization Model

In this section, the mathematical model of the assumed transportation problem is described in a fuzzy environment and then defuzzified. To that end, we aim to provide a clear and mathematically rigorous optimization formulation that maps uncertain profits, supply, and demand information into a computable linear programming model, obeying the same topological constraints as the original transportation network. The objective function is to maximize profit, which is directly related to the imbalanced nature that results from mapping a fuzzy number into a crisp one.

5.1. Sets, Variables, and Parameters

Consider a transportation network consisting of a finite set of supply nodes and a finite set of demand nodes. Let $I = \{1,2,\ldots,m\}$ denote the set of supply nodes, representing production facilities or distribution centers, and let $J = \{1,2,\ldots,n\}$ denote the set of demand nodes, representing markets or consumption points. The decision variable x_{ij} Denotes the quantity shipped from supply node $i \in I$ to demand node $j \in J$. These shipment quantities constitute the core decision variables of the optimization model and are assumed to be continuous and non-negative.

The transport network is uncertain in terms of unit profits, supply capacities, and demand amounts, all of which are initially represented by trapezoidal fuzzy numbers (as developed in the previous sections). These parameters are defuzzified into crisp values, which describe the feasible region and objective function of the optimization problem.

5.2. Defuzzified Parameters

To render the fuzzy transportation problem amenable to mathematical optimization, all fuzzy parameters are transformed into deterministic equivalents using an appropriate defuzzification operator. Let $\tilde{p}_{ij}$, $\tilde{s}_i$, and $\tilde{d}_j$ Denote the trapezoidal fuzzy profit, supply, and demand parameters, respectively. The corresponding defuzzified values are obtained as

$$p_{ij} = R(\tilde{p}_{ij}), \quad s_i = R(\tilde{s}_i), \quad d_j = R(\tilde{d}_j),$$

Where R represents a defuzzification operator. In the proposed framework, the centroid-based operator R_C is employed as the primary transformation mechanism, while the mean-based operator R_M It is used comparatively to assess sensitivity to the choice of defuzzification rule.

This transformation process yields a deterministic transportation network with variables that capture the stability region and confidence bounds from the fuzzy presentation. It is worth mentioning that the defuzzification procedure for profit, supply, and demand is conducted in a manner that guarantees the internal consistency of the obtained optimization model.

5.3. Profit Maximization Formulation

Based on the defuzzified parameters, the transportation problem is formulated as a linear programming model that seeks to maximize total profit. The objective function is given by

$$\max Z = \sum_{i\in I}\sum_{j\in J} p_{ij}\ x_{ij},$$

where p_{ij} Represents the defuzzified unit profit associated with shipping from node i to node j.

The objective function in Eq. (9) is subject to supply and demand balance constraints. For each supply node i, the total quantity shipped cannot exceed the available defuzzified supply, yielding a result of.

$$\sum_{j\in J} x_{ij} = s_i, \quad \forall i \in I.$$

Similarly, for each demand node j, the total quantity received must satisfy the defuzzified demand requirement, leading to

$$\sum_{i\in I} x_{ij} = d_j, \quad \forall j \in J.$$

All shipment quantities must be non-negative.

$$x_{ij} \geq 0, \quad \forall i \in I, \forall j \in J.$$

Together, the previous equations define a standard linear programming transportation model with a profit-maximization objective. The formulation is solved using an exact linear programming solver, ensuring global optimality of the obtained solution.

5.4. Treatment of Unbalanced Networks

In implementation, the defuzzyfication of supply and demand factors may result in a discrepancy between overall available supplies and required demands. This kind of imbalance arises naturally as fuzzy parameters are transformed individually, and the reconciliation of their crisp forms does not imply fulfillment of the equality between total supply and total demand.

This can be solved by balancing the transportation network with a dummy node. If the sum of all defuzzified supply is higher than the sum of all defuzzified demand, a dummy demand node is created with a demand equal to the surplus. If, on the other hand, total demand is greater than total supply, a dummy supply node is added with a flow equal to the deficit. The profit associated with each transportation link to the dummy node is set to zero, ensuring that the addition of these links does not artificially increase or decrease the objective value.

The dummy node, economically, is a measure of the capacity not utilized or the demand that went unmet , depending on the direction of the imbalance. This preserves the structural features of the transportation model and permits solving optimization problems that do not alter the economic interpretation for actual shipment decisions.

6. Computational Framework

This section outlines the computational scheme for solving the fuzzy transportation model and deriving optimal shipment decisions. The framework is designed to support methodological transparency, reproducibility, and exact optimality, while separating uncertainty modeling from optimization. All calculations are carried out using a solver-based method, which ensures that no heuristic or manual techniques were employed and provides consistent results regardless of the shape of the uncertainty.

6.1. Solver-Based Algorithm

The first step of computation is to provide all fuzzy set transportation parameters. The trapezoidal fuzzy numbers are used to represent unit profits, supply capacity, and demand requirements in pessimistic, typical, and optimistic scenarios. Such fuzzy inputs are the sole source of uncertainty in the model, and they remain in their fuzzy form until defuzzification occurs.

Once the fuzzy parameters are characterized, defuzzification is used to convert them into deterministic values that can be optimized. In this paper, the centroid-based defuzzification is considered the key transformation, whereas we use the mean-based method as a reference in specific experiments for investigating the effect of qualifying ranking rules. Such a procedure results in sharp profit coefficients, supply capacities, and demand requirements that capture the information contained in the original fuzzy representations.

After defuzzification, the transportation network is checked for an equilibrium condition between the net total supply and the net total demand. Any imbalance at the specific process-to-process transformation level is compensated by an artificial supply or load boundary node, as explained in Section 3. The introduction of dummy nodes makes the optimization problem feasible, and it does not alter the economic sense of real shipment decisions.

The equilibrium transportation network is further formulated as a linear programming problem, with the profit-maximization model described in Section 5. Here, all variables, objective coefficients, and linear terms of the constraints are organized into a canonical linear programming form. This formulation process is completely deterministic and directly yields the defuzzified network inputs.

By solving the resulting linear program with an exact optimization solver, we can obtain a globally optimal shipment plan, along with its associated maximum profit value. After the entire solver has been executed, the optimal shipment amounts are extracted and evaluated. These findings will serve as the starting point for further comparative and robustness investigations.

Lastly, a sensitivity analysis is performed by varying the uncertainty-related parameters of the trapezoidal plateau (e.g., width and total fuzziness range) and solving the optimization problem at each scenario. The solutions obtained are compared according to the quantitative measures of robustness introduced in Section 4, enabling the quantification of the stability and reliability of the decisions.

6.2. Implementation Details

All computational experiments in this paper are conducted using the LINGO optimization environment. We choose LINGO because of its strong linear programming capabilities, clear syntax for system definition, and the capability to solve large-scale optimization problems with guarantees on optimality. The transportation model is modeled directly as a linear program, and the defuzzified coefficients are provided as numeric data to the Lingo software, with shipment amounts representing continuous variables.

Since a solver-based framework is used, all solutions reported in this work are optimal, satisfying the optimality conditions of the linear programming procedure. Unlike heuristic or approximate methods, the solver ensures convergence to a global optimum if the model is feasible. This property is significant in the context of uncertainty analysis, as it ensures that observed variations in shipment decisions or profit values are due to changes in modeling assumptions, rather than a feature of the solution itself.

The clear distinction between fuzzy modeling, defuzzification, and optimization also enables the reproducibility and extensibility of this framework. Different defuzzification rules or types of uncertainty can be incorporated without altering the optimization structure on which the framework is based, making it suitable for comparative purposes as well as further methodology development.

7. Numerical Experiments

In this section, we present a series of numerical experiments conducted to analyze the performance of our solver-based fuzzy transportation model in controlled uncertain scenarios. The purpose of these experiments is two-fold. First, they offer a first-level validation of the model in a context where uncertainty properties can be precisely observed and understood. Second, they facilitate the exploration of alternative defuzzification rules and their effects on profit values and shipment decisions, without introducing the complexity of real-world data structures. This is achieved by implementing synthetic scenarios, which are utilised before the presentation of the practice case.

7.1. Synthetic Scenarios

Unit profits, supply capacities, and demand requirements are produced as trapezoidal fuzzy numbers in these synthetic experiments, which include direct lower bounds, stability regions, and upper bounds. Varying the widths of the trapezoidal supports and plateaus while holding the network structure fixed introduces varying degrees of uncertainty. By this means, the effect of uncertainty size on optimal profit levels and allocation strategies can be analyzed in a controlled way.

As an example of the impact of defuzzification on profit parameters, Table 1 provides the explicit format of the typically two sets that result when centroid- and mean-based ranking extraction methods are used. The values included in the table show how an alternative defuzzification method can indeed result in considerable differences in the numerical profit coefficients passed to an optimization model, even after two models with the same fuzzy inputs.

Table 1 Defuzzified profit matrices obtained using centroid-based and mean-based defuzzification

Route (i, j)	p_{ij} (Centroid)	p_{ij} (Mean)
(1,1)	365.0	370.0
(1,2)	342.5	345.0
(2,1)	355.0	360.0
(2,2)	330.0	335.0
(3,1)	340.0	345.0
(3,2)	315.0	320.0

While the quantitative disparities between the two matrices may appear modest, their impact on optimal shipment decisions can be magnified due to the optimization process, especially in networks with multiple competing links. This remark justifies the robustness analysis that will be carried out in subsequent sections, where allocation stability is directly assessed.

7.2. Solver Performance

All synthetic examples are solved using an exact linear programming solver, and the numerical performance verifies that our approach is efficient and robust. For the selected sizes of networks and types of uncertainty, the runtimes of the solver are so trivial that no additional computational overhead is introduced by including either fuzzy modeling or defuzzification over a regular transportation problem.

What is more important, the solution stability can be well maintained in repeated runs and with other kinds of defuzzification methods. If shifting decisions occur, these can be unambiguously traced back to differences in the defuzzified parameters, rather than to numerical uncertainties or convergence problems. This characteristic is evidence that the solver-based formulation constitutes a robust framework for uncertainty analysis, and that eventual deviations from the results extracted are indeed representative of modeling effects, rather than due to idiosyncrasies in the solution path.

The knowledge gained from these toy examples constitutes an intermediary step toward applications to concrete cases, as we show in the next section, for the same computational setting applied to a logistics distribution network inspired by an ancient Egyptian transportation system.

8. Case Study: Egyptian Logistics Network

This section presents an example motivated by logistics and fuel distribution networks in Egypt. The case study does not aim to reproduce any particular company, but it illustrates how the developed fuzzy transportation model can be used in a realistic, large-scale logistics context under uncertainty regarding costs, supply amounts, and demand quantities. The application presents itself as a realistic validation of the modeling assumptions, defuzzification technique, and solver-based optimization approach proposed in the previous sections.

8.1. Case Description

Egypt is also an ideal case study for transportation logistics modeling due to its geographically variable landscape, compact consumption areas, and heavy reliance on road-based freight movements. Urban and industrial centers are intermixed with high-marsh distribution pathways serving production and refining facilities to consumption markets. Such features lead to transportation systems with profit, capacity, and demand as quantities under chronic uncertainty, which definitely justifies a fuzzy approach.

We study the problem in a logistics network arising from the distribution of fuel and energy products, where profit variability is due to variations in operating costs, regulated pricing mechanisms, and demand patterns in different regions. The network comprises 4 supply nodes, each representing an important production or storage area, and 5 demand nodes representing the main consumption areas. The meaning of the network is logistical, namely that supply nodes are potential sources of distributable products, whereas demand nodes are aggregated regional markets.

The fuzzy, uncertain supply capabilities and demand constraints are expressed as Trapezoidal Fuzzy numbers. The fuzzy parameters are presented in Table 2, indicating pessimistic, stable, and optimistic operation statuses.

Table 2: Trapezoidal fuzzy supply and demand parameters (in thousand units)

Node	a	b	c	d
Supply 1	180	200	220	240
Supply 2	160	180	200	220
Supply 3	140	160	180	200
Supply 4	120	140	160	180
Demand 1	220	240	260	280
Demand 2	150	170	190	210
Demand 3	110	130	150	170
Demand 4	160	180	200	220
Demand 5	90	110	130	150

Likewise, the Returns of Unit Shipping for all supply–demand pairs are modeled as trapezoidal fuzzy numbers. These earnings reflect variations in newco due to transport distance, operational efficiency, and regional price effects. The fuzzy profit matrix is shown in Table 3.

Table 3 Trapezoidal fuzzy profit matrix p̃_ij

Route	a	b	c	d
S1–D1	320	350	370	400
S1–D2	300	330	350	380
S1–D3	360	390	410	440
S2–D1	340	370	390	420
S2–D4	290	320	340	370
S3–D1	380	410	430	460
S3–D2	350	380	400	430
S4–D4	360	390	410	440
S4–D5	280	310	330	360

8.2. Data Construction

The fuzzy parameters employed in this Case Study are designed to mimic the practical variability of dispatch and fuel distribution systems. Instead of using single-point estimates, each parameter is described by a trapezoidal fuzzy number, which encompasses an enclosed area with plausible values. The lower and upper limits of this plateau indicate that extreme faults can be detected without false alarms and without affecting the standard system, respectively.

This design makes it possible to integrate real-life uncertainty without the need for proprietary and detailed datasets that are usually not available in logistics research. The trapezoidal form has, in this sense, a good compromise between being flexible enough to mimic symmetric and asymmetric uncertainty, and at the same time being interpretable. All fuzzy parameters are created homogeneously for supplies, demands, and profits, thus leading to a coherent and economically interpretable transportation network.

There is no defuzzification at this time. None of the parameters in this model are crisp, and only after they are defuzzified by a defuzzification operator, as described in Section 4, can some parts be plugged into the original optimization model.

8.3. Optimization Results

After defuzzification by the centroid-based method, the transportation network is balanced and solved using the linear programming model mentioned in Section 5. Then, the optimal shipment plan will indicate how much to ship between each pair of locations to achieve the maximum possible profit while adhering to all supply and demand restrictions.

The results of the optimal shipment quantities compared with centroid-based defuzzification are listed in Table 4. We only display routes with positive shipment volumes for clarity.

Table 4 Optimal shipment plan obtained using centroid-based defuzzification

Route	Shipment quantity
S1–D3	140
S2–D1	180
S3–D1	80
S3–D2	90
S4–D4	160
S4–D5	20

To assess the sensitivity of the results to the defuzzification rule, the model is additionally solved using mean-based defuzzification. The obtained total profits and decision stability metrics are presented in Table 5.

Table 5 Profit comparison and decision stability

Method	Total profit	DSI
Centroid-based	247,800	1.00
Mean-based	251,400	0.83

The total profit varies depending on the defuzzification strategies, but most shipment decisions remain unchanged; there's a high Decision Stability Index value. This observation indicates that the algorithm provides robust shipment planning solutions, which are immune to moderate differences in defuzzification rules, and reinforces the fact that the obtained solutions are reliable under uncertainty.

9. Sensitivity and Robustness Analysis

In this subsection, we examine the sensitivity of our developed transportation model to variations in uncertainty characteristics and assess the robustness of the corresponding decisions. The study examines how changes in the formulation of trapezoidal fuzzy parameters affect both total profits and the stability of optimal shipment schedules. By adjusting uncertainty-embedded factors in a structured manner and solving the optimization model, this analysis provides insight into how well our approach can tolerate more complex circumstances in which uncertainties become more challenging.

9.1. Plateau Sensitivity

The first sensitivity analysis considers the effect of the ridge width of trapezoidal fuzzy numbers in relation to optimal profit. The plateau corresponds to the complete plausible interval and measures the stability of the actual parameter. By enlarging or reducing this area and maintaining the same overall support for the fuzzy number, one can assess how sensitive the optimization results are to the assumed stability of profits, supplies, and demands.

In this way, a series of scenarios is produced in which the width of the plateau is made successively larger, for which the range considered entirely plausible slowly expands as well. For each testing case, the fuzzy parameters are defuzzified using the centre of gravity-based method, and the transportation network is balanced ; its linear programming model is then solved. Then, the corresponding solution profits are computed and compared among different situations.

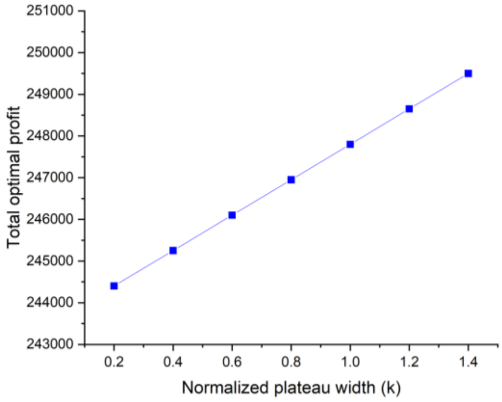

Figure 2: Effect of plateau width on the total optimal profit

The relationship between total profit and plateau width is shown in Figure 2. It can be observed that moderate changes in the plateau width result in only minimal changes in total profit, demonstrating that the optimal solutions are not highly sensitive to the exact definition of the stability interval. This can be interpreted as the balancing property of centroid-based defuzzification, which emphasizes the center value of a fuzzy number while considering uncertainty at its extensions. From a decision-making perspective, this finding indicates that the model produces profit predictions that remain stable when one (reasonably) varies the confidence in parameter values.

9.2. Uncertainty Expansion

To further test robustness, another experiment is conducted to consider further enlarging the entire uncertainty interval of the fuzzy parameters. By uncertainty expansion, we mean the increase in the distance between the lower and upper bounds of the trapezoidal fuzzy number, while preserving their relative position in the plateau region. This process models situations where there is an increase in external volatility, such as economic crises and fluctuating operations.

For every uncertainty expansion level, the disjunction is solved, and the fuzzified transportation model results are tested against a baseline solution prior to defuzzification using the Decision Stability Index discussed in Section 4. The findings suggest that decision stability diminishes as uncertainty becomes greater, in line with the inherent sensitivity of allocation decisions to extreme parameter drifts. Despite this, the loss is limited over a wide range of uncertainty degrees, indicating that the core structure of the optimal shipment plan can still be preserved when significantly more fuzziness is introduced. This result demonstrates the reliability of the presented solver-based approach and shows that considering trapezoidal fuzzy modeling does not lead to erratic or unstable decision rules.

The sensitivity analysis and validation tests empirically demonstrate a certain level of reliability for the proposed approach. They also demonstrate that not only the profit results but also the shipment decisions reveal a significant robustness against uncertainty specification, which further validates the model's finality in practical transportation and logistics planning.

10. Discussion

The results drawn from the previous analyses provide several methodological and practical insights into modeling urban transportation network design problems under uncertainty. They specifically demonstrate how trapezoidal fuzzy modeling, combined with solver-based optimization, enhances both the practicality and reliability of profit-optimized transportation decision-making.

The employment of trapezoidal fuzzy numbers enables uncertainty to be better represented compared to the traditional triangular form. In the real world, many logistics system parameters (e.g., profits, capacities, and demands) are not focused on a single most likely value but fluctuate within a range to ensure safe operation. The plateau nature of the trapezoidal fuzzy numbers intuitively reflects this property, as it permits a full-membership span during which all values are equally credible. This property enables the model to distinguish between stable states and extremes, making it less prone to overreacting to marginal uncertainty. The sensitivity analysis also indicates that the observations made on how decisions obtained from trapezoidal models are less sensitive to variations in small parameters confirm the relevance of this modelling approach towards practical applications.

Moreover, the use of a solver-based optimization method employed in this work brings an additional level of rigor and confidence to our findings. The reference approach is based on representing the transportation problem as a linear program and solving it with an exact optimization solver to ensure that the solutions obtained are globally optimal. This characteristic may be particularly valuable in uncertainty-dominated analyses, since the vicinity of parameter values can produce different local optima when heuristic or approximate optimization techniques are employed. Indeed, the consistency of shipment decisions across alternative defuzzification rules and uncertainty settings implies that our solver-based methodology provides a solid foundation for comparative and robustness investigations.

In terms of managerial implications, the proposed framework has several practical benefits. Logistics and transportation planning officials often need to develop resource allocation plans based on uncertain market conditions to ensure profitable operations and maintain operational capability. The fact that uncertainty can be directly incorporated using trapezoidal fuzzy parameters offers an option for checking the obtained decisions against pessimistic, best, and non-melting scenarios using a single model. Additionally, the measures of robustness proposed in this paper provide new decision-stability metrics for managers to evaluate not only the anticipated levels of profit but also the trustworthiness of allocation plans. Accordingly, the framework facilitates explicit decision-making by trading off profits against robustness to uncertainty in these rapidly changing and volatile logistics environments.

These discussions suggest that the developed approach is not only an extension of theoretical models in fuzzy transportation but also a valuable method for practical decision-making support in analyzing complex and realistic transportation networks.

11. Conclusions and Future Work

A solver-based approach is proposed in this paper for profit maximization in unreliable transportation networks. The trapezoidal fuzzy modeling, combined with exact linear programming, in the proposed model can effectively address the shortcomings of conventional transportation models in encoding uncertainty, achieving solution reliability, and conducting robustness analysis. The model represents diversity in revenues, production capacity, and demand needs while maintaining a transparent and interpretable approach to optimal solutions.

The use of trapezoidal fuzzy numbers enables an explicit description of stability intervals that are often present in real logistic systems and cannot be accurately modelled by simpler representations of uncertainty. This capability enables the framework to differentiate between normal and extreme operating conditions without relying excessively on marginal departures. The defuzzification approach and the systematic balancing process for unbalanced nets ensure that uncertainty is clearly and consistently integrated into the nets before optimization.

The solver-based approach ensures the global optimality of the solutions found and eliminates ambiguity associated with heuristics or approximations. Experimental results, as well as the case study of Egyptian logistics, illustrate that the proposed model yields stable shipment decisions along with reliable profit consequences, ensuring sustainable development. It can work effectively under different defuzzification rules and varying degrees of uncertainty. The SAs and RAs also provide evidence that both profits and allocations derived from the model are stable to reasonable variations in uncertainty specification, which further stresses the practical feasibility of our model.

Several future research directions will naturally follow from this work. The proposed approach can be generalized for a multi-objective transportation model, in which profit, service level, and risk factor are considered simultaneously. Environmental issues, such as carbon dioxide ($CO2$) emissions and energy consumption, can also be addressed to facilitate environmentally friendly logistics planning. Finally, hybrid methodologies that combine fuzzy and stochastic representations of uncertainty appear to be a promising approach for identifying epistemic and aleatory uncertainty in transportation systems. Such extensions would strengthen the decision-support potential of the framework and extend its relevance to more complex and dynamic logistics situations.

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